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ON A GRAPHIC METHOD OF COMPARING THE RELATIVE EFFICIENCIES OF DIFFERENT SPECTROSCOPES.

By H. C. Lord.

Anyone who has occasion to use a spectroscope of comparatively small resolving power, in order to obtain photographic spectra, at once finds that, with rapid coarse-grained plates, it is absolutely impossible to photograph nearly as many lines in the solar spectrum as can be observed visually, and that with fine-grained lantern slide plates much better results can be obtained. The difficulty is evidently due to the coarse granular structure of the rapid plate. Thus far the makers of photographic plates have been unable to secure sensitiveness and at the same time fineness of grain. It would seem proper therefore that in the construction of a spectroscope which is to be used photographically, the design of its optical parts should be carried out with special reference to the character of the plates to be used. A moment's consideration shows that, since the linear dimensions of the diffraction pattern of the image of a straight line formed by an objective vary directly with its focal length, while the size of

the grain of the plate remains constant, the ratio of these two may be made anything desired simply by changing the focal length of the camera, and thus what might be called the effective size of the grain be reduced almost without limit. In order to maintain the brightness of the spectra it will only be necessary to reduce the dispersion of the prism train employed. This, of course, reduces the resolving power of the instrument, but as already the optical resolution is far in excess of that of the plates this would seem at first sight to be of no disadvantage.

In order to investigate more thoroughly than I had yet done these several conditions, to explain moreover my inability to photograph the faint iron lines of the first type stars with the spectrograph of the Emerson McMillin Observatory, and to see if it were not possible by simple changes in its optical design so to modify the instrument that these lines could be photographed and yet the brightness of the spectra be unimpaired, I offered a course of lectures to the students of the Ohio State University on the theory of the spectroscope and its design. Though much has been recently written on the subject, notably by Professor Wadsworth[1] to whose able papers I was indebted for most of the material of the lectures above referred to, it has seemed to me that previous investigators have confined their attention to bright line spectra and I was unable to find a single instance where the case, most frequently met with in nature, of dark absorption lines on a background of continuous spectra was made the basis of the discussion. This may perhaps have been due to the limited library facilities at my command, but I developed a graphical method of treating this problem, which, though not strictly rigorous, is very general in its application, and not only furnishes an accurate and reliable guide for the comparison of the efficiencies of different instruments but shows at a glance the results of varying any one of the several elements which enter into their optical design.

[1] WADSWORTH, "The Modern Spectroscope," XVIII, this JOURNAL, May 1896. "On the Conditions of Maximum Efficiency in Astrophotographic Work," this JOURNAL, August 1897. "On the Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Memorie della Societa degli Spettroscopisti Italiani*, Vol. XXVI, 1897.

This method has been so useful to me that I thought it might be of interest to others, and venture to offer it to the readers of the ASTROPHYSICAL JOURNAL.

In his article, "Wave Theory," in the *Encyclopædia Britannica*, Lord Rayleigh gives expressions for the intensity at any point ξ of the diffraction pattern of an infinitely narrow line formed by an objective of either a rectangular or circular aperture. These expressions are equivalent to the following:

$$(1)\ I_R = C\frac{\sin^2\zeta}{\zeta^2} \text{ and } (2)\ I_C = C\zeta^{-3}K_1(2\zeta),$$

where I_R and I_C are the intensities for rectangular and circular aperture, respectively, and

$$\zeta = \frac{2\pi R\xi}{\lambda f};$$

$2R$, f and λ being the width of the rectangular or the diameter of the circular aperture, the focal length of the objective and wave-length of light respectively. The constant C contains both R and f, but as I will express intensities in terms of the intensity at the point $\zeta=\xi=0$, leaving to geometrical optics the determination of the absolute intensities, these factors may be neglected. The function $K_1(Z)$ is given by Lord Rayleigh in two forms, namely:

$$K_1(Z) = \frac{2}{\pi}\left\{\frac{Z^3}{1^2\cdot 3} - \frac{Z^5}{1^2\cdot 3^2\cdot 5} + \frac{Z^7}{1^2\cdot 3^2\cdot 5^2\cdot 7} - \frac{Z^9}{1^2\cdot 3^2\cdot 5^2\cdot 7^2\cdot 9} + \text{etc.}\right\} \text{ and}$$

$$K_1(Z) = \frac{2}{\pi}\left\{Z + Z^{-1} - 3Z^{-3} + 1^3\cdot 3^2\cdot 5Z^{-5} - 1^2\cdot 3^2\cdot 5^2\cdot 7\ Z^{-7} + \text{etc.}\right\}$$

$$-\sqrt{\frac{2Z}{\pi}}\cos\left(Z-\frac{1}{4}\pi\right)\left\{1 - \frac{(1^2-4)(3^2-4)}{1.2.(8Z)^2} \cdots\right\}$$

$$-\sqrt{\frac{2Z}{\pi}}\sin\left(Z-\frac{1}{4}\pi\right)\left\{\frac{1^2-4}{1.8Z} - \frac{(1^2-4)(3^2-4)(5^2-4)}{1.2.3(8Z)^3}\right\}$$

From these formulae I have computed the values of I_R and I_C for values of ζ and from 0 to 13.50 for every 0.25. These values are given in Table I.

TABLE I.

ζ	I_R	I_C	ζ	I_R	I_C	ζ	I_R	I_C
0.00	1.0000	1.0000	4.75	0.0443	0.0442	9.50	0.0001	0.0064
0.25	0.9793	0.9834	5.00	0.0368	0.0420	9.75	0.0011	0.0057
0.50	0.9194	0.9365	5.25	0.0268	0.0375	10.00	0.0030	0.0056
0.75	0.8260	0.8594	5.50	0.0165	0.0314	10.25	0.0050	0.0061
1.00	0.7081	0.7620	5.75	0.0078	0.0249	10.50	0.0070	0.0064
1.25	0.5764	0.6508	6.00	0.0022	0.0191	10.75	0.0081	0.0069
1.50	0.4422	0.5334	6.25	0.0000	0.0147	11.00	0.0083	0.0072
1.75	0.3162	0.4199	6.50	0.0011	0.0120	11.25	0.0074	0.0074
2.00	0.2067	0.3151	6.75	0.0044	0.0110	11.50	0.0058	0.0072
2.25	0.1196	0.2247	7.00	0.0088	0.0114	11.75	0.0038	0.0065
2.50	0.0573	0.1522	7.25	0.0129	0.0125	12.00	0.0020	0.0057
2.75	0.0193	0.0985	7.50	0.0156	0.0139	12.25	0.0006	0.0048
3.00	0.0022	0.0621	7.75	0.0165	0.0148	12.50	0.0000	0.0041
3.25	0.0011	0.0420	8.00	0.0153	0.0150	12.75	0.0002	0.0036
3.50	0.0100	0.0333	8.25	0.0125	0.0145	13.00	0.0010	0.0032
3.75	0.0232	0.0325	8.50	0.0088	0.0132	13.25	0.0023	0.0034
4.00	0.0358	0.0360	8.75	0.0051	0.0114	13.50	0.0035	0.0036
4.25	0.0443	0.0403	9.00	0.0021	0.0094			
4.50	0.0472	0.0440	9.25	0.0004	0.0077			

From this table the curves given in Fig. 1 and Fig. 2 were drawn. From these two curves it is possible to construct graphically the form of the diffraction pattern of the image of a slit either bright on a dark background, or dark on a bright background, either seen directly or through a prism; provided only the slit is long in comparison with its width, which is the case of all solar spectroscopes. The case where the slit is replaced by the diffraction image of a star, as occurs in all stellar spectroscopes except the objective prism, I have not yet succeeded in reducing to formulae which are practically applicable. In what follows I shall confine myself entirely to the case of circular aperture. The case of rectangular aperture is exactly the same, it only being necessary to start with curve (1) in place of curve (2).

Case 1.—A bright slit of angular width σ on a dark background. This case has been treated analytically by Wadsworth[1] and we have

[1] See previous footnote, also other papers by Wadsworth.

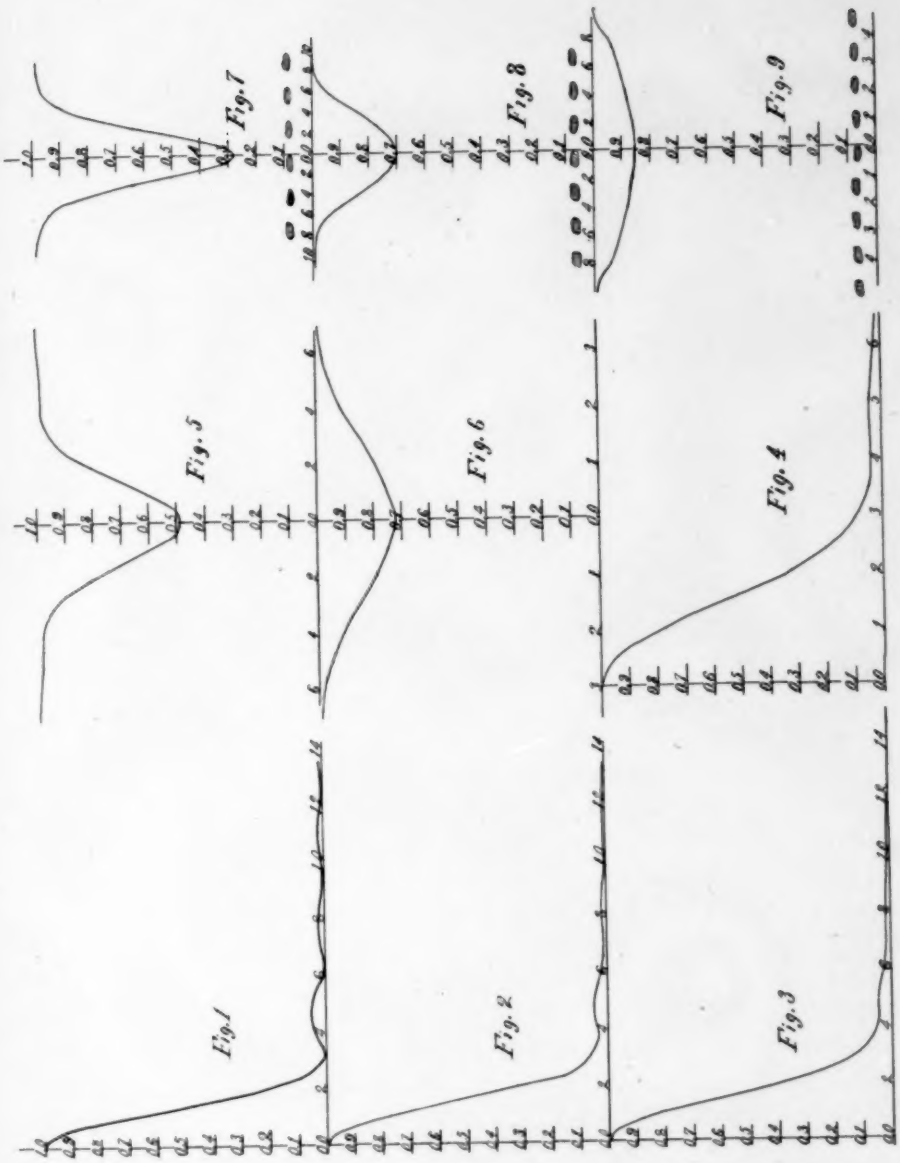
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9

$$I_1 = C' \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta, \qquad (3)$$

where $\zeta_0 = \frac{2\pi R \sigma}{\lambda}$.

This integral cannot be obtained directly. Wadsworth evaluated by mechanical quadrature. This can be done much more easily and rapidly by means of the planimeter as follows: the ordinate of the curve (3) at any point ζ is evidently proportional to that portion of the curve (2) included between the ordinates whose abscissae are $\zeta - \frac{1}{2}\zeta_0$ and $\zeta + \frac{1}{2}\zeta_0$; hence in order to plot the curve (3) it is only necessary to measure with a planimeter the area above specified and draw an ordinate at the point ζ proportional to this area. In practice it is convenient to take as unity the area between the ordinates at $0 - \frac{1}{2}\zeta_0$ and $0 + \frac{1}{2}\zeta_0$ as this gives the intensities in terms of the intensity at the origin. In the absence of the planimeter, if the several curves be drawn on cross-section paper the areas can be obtained by counting the number of squares included between any two ordinates. For example, a slit whose width is 0.004 mm at the focus of a collimator of a focal length $f = 380$ mm and aperture $= 25.3$ mm is illuminated by monochromatic light, $\lambda = 4340$. To find the form of the diffraction pattern formed by an objective of 25.3mm aperture. Here $\zeta_0 = \frac{2\pi \times 12.65 \times 0.004}{0.000434 \times 380} = 2$ nearly. The resulting curve is given in Fig. 3. Fig. 4 gives a similar curve where the aperture is reduced one half. In plotting these curves it should be noted that since ζ contains the factor R, the scale of ζ is taken twice as great in Fig. 4 as in Fig. 3.

Case 2.—A dark line of angular width σ on a bright background. This case has been analytically investigated by Michelson[1] for rectangular aperture. It differs from the preceding case obviously only in the limits which are to be assigned to the integral. Hence we have

[1] MICHELSON, "On the Limit of Visibility of Fine Lines in a Telescope," this JOURNAL, June 1895.

$$I_2 = C' \int_{\zeta + \frac{1}{2}\zeta_0}^{\infty} \zeta^{-3} K_1(2\zeta)\, d\zeta + C' \int_{-\infty}^{\zeta - \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta, \qquad (4)$$

which reduces to

$$I_2 = C' \int_{-\infty}^{+\infty} \zeta^{-3} K_1(2\zeta)\, d\zeta - C' \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta. \qquad (5)$$

The first integral of expression (4) is the total area of the curve (2), but this may be taken without appreciable error as being equal to the total area of the curve only as far as given in the table which accompanies this article. In the case of a rectangular aperture the first integral reduces to $C\pi$, and it will be found that if the area of a square block whose height is unity and whose base is π be measured with the planimeter, the area thus formed will differ from the total area of curve (2) as given above by less than the error of measurement. The second integrals can be evaluated exactly as in the preceding case, except that the total area of the curve is to be taken as unity. Curves (5) and (6) were drawn from the data given in the above example, the slit being replaced by a wire of the same diameter on a bright background of light, $\lambda = 4340$. An inspection of these two curves shows that a decrease of aperture causes not only a decrease in resolving power but a decrease in contrasting power as well. This point was brought out by Michelson in the paper above quoted.

To test these conclusions experimentally I covered the lens of a camera with a very narrow rectangular cap and photographed a drawing made of very fine horizontal and vertical lines, and found it was perfectly possible to cause either the vertical or horizontal lines to disappear from the resulting photograph, while the lines at right angles to them remained as sharp and distinct as when photographed with the full aperture of the lens.

In the discussion of the two cases given above I have in a large measure repeated the work of Wadsworth and Michelson, but have done so for the sake of completeness and to illustrate

the use of the planimeter and the graphic method of solving these problems. The case next to be considered has, so far as I am aware, never heretofore been discussed, and I shall treat it, therefore, somewhat more in detail than I have the preceding ones.

Case 3.—A slit of angular width σ, illuminated by light of all wave-lengths except those between λ_1 and λ_2 ($\lambda_2 - \lambda_1 = \Delta\lambda$ being a small fraction of λ) is viewed through a telescope in front of which is placed a battery of one or more prisms or a grating.

This is approximately the case of the Fraunhofer lines. They are represented more closely by two laws[1] proposed by Maxwell and Michelson for the distribution of light in a normal source. One, $f(\Phi) = e^{-k\zeta^2}$, contains a constant K, which varies with the nature and condition of the body emitting the light, the other, $f(\Phi) = e^{\frac{\sin^2 r\zeta}{\zeta^2}}$, contains a similar factor r. Thus far I have been unable to discuss this problem when these expressions are introduced. Moreover, since the laws are not yet definitely known, and since very few experimental data are available for the determination of the constants, which vary not only with different substances but with the same substance under different conditions, it seems to the writer not only justifiable but even better to base the comparison of the efficiencies of two spectroscopes upon conditions which, though somewhat arbitrary, are perfectly definite, except in cases where an instrument is to be used for investigations of a particular substance under certain specified conditions, and even in that case the laws given above are unfortunately not too well established.

It has been already shown that the distribution of intensity in the diffraction image of a slit of angular width σ is given by the equation

$$I_1 = C' \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta$$

[1] RAYLEIGH, *Phil. Mag.*, April 1889, p. 298; also Michelson, *Phil. Mag.*, September 1892, and this JOURNAL, November 1895, p. 251.

and that this curve can be readily drawn from curve (2) with the aid of a planimeter. Light of each and every wave-length, except those already noted as omitted, gives rise to such a curve. These curves would all be superimposed were it not for the dispersive power of the prism, whose action is to shift each one of these curves along the axis of ζ.

Let us take as the origin of coördinates a point in the focal plane which would be occupied by the center of the geometrical image of the slit when illuminated by monochromatic light of wave-length $\lambda_0 = \frac{1}{2}(\lambda_1 + \lambda_2)$. The wave-length of light corresponding to any other point ζ would be given by the equation (5) $\lambda = \lambda_0 + m\zeta$, where m is a constant depending upon R, f, and the dispersive power of the prism train employed, and whose value can be predetermined from the known constants of the glass out of which the optical parts of the spectroscope are to be made. This equation only holds for a small distance from the origin of coördinates, a distance large, however, in comparison with the dimensions of the diffraction pattern. Let I_{III} be the intensity at any point ζ of the resulting image; then evidently

$$I_{III} = C'' \int_{\lambda=\lambda_2}^{\lambda=\infty} I_1 \, d\zeta + C'' \int_{\lambda=0}^{\lambda=\lambda_1} I_1 \, d\zeta.$$

This is equivalent to

$$I_{III} = C'' \int_{\lambda=\lambda_4}^{\lambda=\infty} I_1 \, d\zeta + C'' \int_{\lambda=0}^{\lambda=\lambda_3} I_1 \, d\zeta + C'' \int_{\lambda=\lambda_3}^{\lambda=\lambda_4} I_1 \, d\zeta - C'' \int_{\lambda=\lambda_1}^{\lambda=\lambda_2} I_1 \, d\zeta. \quad (6)$$

An inspection of curve (3) shows that the limits λ_3 and λ_4 may be so chosen that light of wave-lengths greater than λ_4 and less than λ_3 will produce little or no effect at a distance from the origin large in respect to the size of the diffraction pattern, and yet at the same time $\lambda_4 - \lambda_3$ will be small in comparison to λ. In other words, the first two integrals may be considered equal to zero over an extent of the focal plane more than sufficient to include the entire effective part of the diffraction pattern of the

dark Fraunhofer line. Expressing the limits of (6) in terms of ζ through equation (5) we have

$$I_{III} = C''' \int_{\frac{\lambda_0 - \lambda_4}{m}}^{\frac{\lambda_3 - \lambda_0}{m}} d\zeta \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta$$

$$- C''' \int_{\zeta - \frac{1}{2}\frac{\Delta\lambda}{m}}^{\zeta + \frac{1}{2}\frac{\Delta\lambda}{m}} d\zeta \int_{\zeta - \frac{1}{2}\zeta_0}^{\zeta + \frac{1}{2}\zeta_0} \zeta^{-3} K_1(2\zeta)\, d\zeta . \quad (7)$$

Since ζ contains the factor $\frac{1}{\lambda}$ the limits of the first integrals are themselves variables and hence these expressions could not be evaluated graphically were it not for the fact that $\lambda_4 - \lambda_3$ is always small in comparison with λ; $\lambda_4 - \lambda_3$ never exceeds $1\,\mu\mu$. That being the case it is evident that no sensible error will be introduced if they are considered constant during the entire second integration, and may be represented graphically by the curve (3) drawn for $\lambda = \lambda_0 = \frac{1}{2}(\lambda_1 + \lambda_2)$. The first of the double integrals expresses the total area of this curve, which may be considered as extending only a finite distance from the origin, and as before, its value determined with the planimeter. The second expresses the area of this curve between two ordinates at the points $\zeta - \frac{1}{2}\frac{\Delta\lambda}{m}$ and $\zeta + \frac{1}{2}\frac{\Delta\lambda}{m}$, and can be found as in the preceding cases. Thus, using curve (2) to derive curve (3), and this latter for curve (7), the problem is completely solved. In order to illustrate this case I will take the case of a line at $\lambda = 4340$ and $\Delta\lambda = 0.086$ Ångström units, and draw the form of the diffraction image as seen in each of three spectroscopes using a slit-width of 0.012 mm, slightly less than would ordinarily be used in practice. The first is the Mills spectrograph of the Lick Observatory, as used by Professor Campbell for the determination of motions in the line of sight. The second is the two-prism spectrograph of this Observatory, and the third is the same instrument when using only one prism and a camera of double focal length. The data are given below.

	I	II	III
	mm	mm	mm
Focal length collimator	722.4	380.	380.
Focal length camera	405.5	380.	760.
Effective aperture	37.4	25.3	25.3
Number of prisms	3.	2.	1.
Slit-width	0.012	0.012	0.012
Number of Ångström units corresponding to $\xi = 1$ mm	12.4	20.9	20.9
ζ_0,	4.5	6.	6.
m,	0.0186	0.043	0.086
$\frac{\Delta\lambda}{m}$,	4.6	2.0	1.0

The data for the Mills spectrograph are taken from Professor Campbell's paper published in the ASTROPHYSICAL JOURNAL, Vol. VIII, No. 3. The focal length of the double camera lens is taken as the focal length. The focal length of the triple lens is not given, but from statements made later in the paper I assume they are practically the same. The value 12.4 was computed roughly from Table I, page 143. Figs. 7, 8, 9, give the intensity curves for these three instruments drawn by the above method. If we assume the average size of the grain[1] of the photographic plate to be 0.025 mm the corresponding value of ζ, for case II, is 1.2. In order to render these three curves strictly comparable it will be necessary to vary the scale of ζ so that the scale of ξ shall be the same in each case; thus the scale of ζ, for case III, is twice that of case II; when this has been done the actual linear value of ζ corresponding to the size of the silver grain will be the same in each case. Below each curve I have drawn to scale the silver grains, leaving a space between them of the same size as the grain itself.

The inspection of these curves shows that there are three important factors which determine the efficiency of a spectrograph, namely, the optical resolving power, the contrasting power, and what may be called the effective size of the grains of the plate. I know of no data sufficient to determine the relative

[1] WADSWORTH, "Efficiency of the Spectrograph," this JOURNAL, May 1896, p. 328.

importance of these three factors, but I hope in the near future to secure a number of photographs of the solar spectrum taken with a spectroscope in which the focal length of the camera can be varied from six inches to four feet, and the dispersive power of the prisms from a single 60° light flint prism, through two dense 60° prisms up to a 14000 line grating. It seems to me that a collection of such photographs taken in connection with the corresponding intensity curves will form a much better basis for the correct design of a spectrograph than the ordinary hard and fast formulae for purity and resolving power, as such curves enable one to see at a glance the effect of varying any one of the several elements which enter into the spectroscope design.

As this paper is already too long I shall not take up the case of double lines and the case of a slit illuminated by light between wave-lengths λ_1 and λ_2. Their graphical treatment is obvious from what has already been given.

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OHIO STATE UNIVERSITY,
March 10, 1899.

PERTURBATIONS OF THE LEONIDS.[1]

By G. JOHNSTONE STONEY and A. M. W. DOWNING.

WHEN the present investigation was undertaken, our knowledge of the perturbations of the Leonids was due to an investigation carried on thirty years ago by Professor J. C. Adams.[2]

His object was to compute the shift in the nodes of the meteoric orbit due to perturbations, and to compare the calculated amount with the amount which had been deduced by Professor Hubert A. Newton from observations made at intervals during the last 1000 years.[3]

For Professor Adams' purpose the perturbations to be computed were the average perturbations; and he accordingly employed Gauss' method, in which the mass of the disturbing planet is supposed to be distributed round its orbit in quantities proportional to the time that the planet occupies in traveling over each portion of its course. This elegant method furnishes the average amount of each perturbation on the supposition that the periodic times of the disturbed body and of the disturbing planet are incommensurable, so that in the course of time the two bodies present themselves in every possible position to one another.

This condition, however, has been but imperfectly fulfilled within the limited period of 1000 years over which the recorded observations extend, especially in the case of the three planets which influence the Leonids most, and indeed are almost the only planets whose attraction needs to be taken into account. These are Jupiter, Saturn, and Uranus: A comparison of the periodic times shows that fourteen revolutions of Jupiter approximate in duration within about one fifth of a year, to five revolu-

[1] Read before the Royal Society.

[2] *Comptes Rendus*, March 25, 1867, p. 651; and for a fuller account see *Monthly Notices*, April 1867, p. 247; or *Monthly Notices*, March, 1897, p. 387, where the last mentioned paper is reprinted.

[3] *Silliman's Journal* 37, 377, 1864; and 38, 53.

tions of the meteors; two revolutions of Uranus occupy about one and three quarters of a year more than this same time, and nine revolutions of Saturn correspond within a fraction of a year to eight revolutions of the meteors.

These cycles have been several times repeated within the period over which the observations extend; and one consequence of these cycles is that there have been oscillations in the rate of the advance of the node about its mean value, so that the times for the showers assigned by applying to the orbit the average shift of its node have usually differed by several hours from the actual times. On one occasion—in A. D. 1533—the shower anticipated the computed time by about twenty six hours, and, as the present investigation shows, a deviation of comparable amount and in the opposite direction is to be expected this year. Accordingly, even if our sole object were to enable astronomers in future to predict more satisfactorily the times of the greater Leonid showers, it would be necessary to prepare for the task by first studying the actual amount of the peturbations in each revolution, and moreover, for meteors occupying various stations along the stream.

For, in fact, the perturbations have not only differed in different revolutions, but even within a single revolution, the meteors which occupy successive positions in the procession are differently affected by the surrounding planets, as is confirmed by the definite results which Herr Berberich has obtained by assuming successively two epochs for the perihelion passage.[1] The dense part of the stream, with which we are chiefly concerned, and which we may call the ortho-stream,[2] is now so long that it takes

[1] See his paper on the perturbations since 1890 of the orbit of the comet which is associated with the Leonids, *A. N.*, No. 3526.

[2] In order to facilitate the study of the Leonids it is convenient to distinguish between a great body of them — the ortho-Leonids — which are traveling round the Sun in nearly identical orbits, and another class of Leonids which we may call clino-Leonids, that are pursuing courses which differ in a more considerable degree from the ortho-orbit. By the ortho-orbit is to be understood the mean of the orbits of the ortho-Leonids.

The ortho-Leonids at present form a compact stream of such a length that it takes

between two and three years to pass each point in its orbit, so that the configurations in which the several parts are presented to the disturbing planets are markedly different. Accordingly, perturbations must have produced in this long stream both sinuosities and an unequal distribution of density;[1] and the first step towards increasing our acquaintance with these and other kindred phenomena, as well as towards gaining a better insight into the past history of the swarm, is to aim first at securing a more intimate knowledge of the perturbations.

With this end in view it was decided, as a first step, to compute the actual perturbations of a definite part of the stream over the whole of one revolution, taking that part of the ortho-stream of which Adams had determined the orbit, and extending the computation over the revolution from the date of the great shower of November 1866, until that day in January 1900, when the same part of the stream will return to the Earth's orbit.

Adams' calculation was based on determinations of the radiant point which were made in 1866, before photography had lent the aid to astronomy which it now yields. Moreover, the circumstance that the Earth deflected the meteors which were then observed by an amount which varied as the shower progressed, was not at that time attended to by observers. Owing to these imperfections, there is a considerable probable error in the mean of the determinations which were made in 1866, and a corresponding uncertainty in the values of the elements computed from that mean. We are accordingly only justified in employing Adams' orbit as approximate. But, fortunately, an error in the orbit, of such an amount as is at all likely to exist, will not

nearly three years to pass each point of its orbit, and so narrow that when the Earth passes obliquely through it the transit occupies only some five or six hours; whereas the clino-Leonids form a less dense and wider stream, which has spread itself the whole way round the ring, and which produces in every November, when the Earth passes through it, a feeble meteoric shower that lasts for several days.

[1] One consequence of the existence of irregularities in the stream of ortho-Leonids is that the ortho-orbit at one cross-section of the stream (*i. e.* the mean of the orbits of the meteors occupying that situation in the stream) is in general not absolutely identical with the ortho-orbits at other cross-sections.

materially affect the perturbations of the orbit, which are what we have at present in view.

The main stream of Leonids—the ortho-stream—is narrow and very long, and it is convenient to divide it into segments, each of which shall be of moderate length. Through one of these, which we may call segment A, the Earth passed in November 1866, and on that occasion there was withdrawn from it that small portion which consisted of meteors which either encountered or passed close to the Earth. Those that actually plunged into the Earth's atmosphere were destroyed: those that passed near were deflected, and were also either accelerated or retarded, and they thus became clino-Leonids. It is with the great majority of the meteors in segment A, which escaped both these fates and continued to be ortho-Leonids, that Adams' investigation is concerned. He ascertained their orbit; and starting from the elements of the orbit as determined by him, the actual perturbations which it has since undergone have been computed, and the main results thus arrived at are embodied in the following table.

As already stated, the calculation has been extended over an entire revolution of that portion of the stream which we have called segment A; and in computing the perturbations, account has been taken of the attraction exercised upon these meteors by Mars, Jupiter, Saturn, and Uranus. At first Venus and the Earth were included, but as the influence of these planets was found to be insensible, they were omitted from the latter part of the calculation.

The expense of carrying on the work has been met partly out of the Government Grant administered by the Royal Society, and partly out of the Royal Society's Donation Fund. The computations have been made by Messrs. F. B. Cooper, J. H. Bell and W. H. Walmsley, members of the staff of the *Nautical Almanac* office. We are also indebted to Mr. E. Roberts, the chief assistant, for his aid in various parts of the work. The method adopted was that by mechanical quadratures, the determinations of the variations of the elements being made at inter-

PERTURBATIONS OF THE ELEMENTS OF THE ORBIT OF SEGMENT A OF THE ORTHO-STREAM IN CERTAIN SELECTED INTERVALS OF TIME. THE ELEMENTS ARE REFERRED TO THE MEAN EQUINOXES OF THEIR RESPECTIVE EPOCHS.

		Elements of the osculating ellipse on 1866, November, $13^d\ 13^h$, as found by Adams	Perturbations of the elements in the selected intervals					Computed values of the elements on 1900, January, $27^d\ 15^h$
			I	II	III	IV	V	
Mean longitude in orbit	ϵ	58° 10.2′	− 4.83′	− 0.32′	−27.98′	−13.99′	−0.70′	58° 34.4′
Longitude of perihelion	π	58° 19′	− 5.37′	+10.70′	− 6.47′	− 4.75′	−0.60′	58° 40.6′
Long. of node (descending)	ν	51° 28′	+29.33′	+ 7.15′	− 1.69′	+70.83′	+0.09′	53° 41.8′
Inclination	i	16° 46′	+11.92′	− 1.01′	+ 1.43′	−28.60′	−0.01′	=16° 29.7′
Angle of eccentricity	ϕ	64° 46.8′	− 3.39′	− 1.70′	+12.06′	+ 7.65′	+0.32′	=65° 1.7′
Mean distance	a	10.3402	+0.015 660	−0.021 271	+0.033 726	+0.038 258	+0.001 747	=10.408 32
Daily motion of ϵ	n	−1.778 57′	+0.004 069′	−0.005 481′	+0.008 678′	+0.009 763′	+0.000 441′	=−1.761 10′

I is the interval from 1866, November 13, to 1871, May 3. In this interval segment A of the ortho-stream crossed the orbits of Jupiter and Saturn.

II is the interval from 1871, May 3, to 1894, December 28. In this interval it crossed the orbit of Uranus, both on the outward and homeward journeys.

III is the interval from 1894, December 28, to 1897, December 30. In this interval it recrossed the orbit of Saturn.

IV is the interval from 1897, December 30, to 1899, May 18. In this interval it recrossed the orbit of Jupiter.

V is the interval from 1899, May 18, to 1900, January 27. This interval brings segment A of the stream back to its descending node.

vals of thirty-six days, except for the period from May 1871 to December 1894, during which time the perturbations were small and progressed so regularly that it was found sufficient to make the computations at intervals of 216 days.

The most noteworthy features are a near approach to Saturn in April 1870, and a near approach to Jupiter in August 1898, at which latter time the meteors in segment A of the stream were at a distance from the planet of only 0.9 of the mean radius of the Earth's orbit. The consequences of these near approaches are brought out in the table. Uranus produced but little effect in this revolution. The planet was at a distance when the swarm crossed his orbit. And the influence of Mars was trifling. So that nearly the whole of the perturbations during this revolution have been caused by Jupiter and Saturn.

The following were the adopted masses of the disturbing planets:

Mars	$\frac{1}{3{,}093{,}500}$
Jupiter	$\frac{1}{1{,}047.879}$
Saturn	$\frac{1}{3{,}501.6}$
Uranus	$\frac{1}{22{,}756}$

In consulting the table, it has to be borne in mind that ϵ, which is there designated, in compliance with the usual convention amongst computers, the "mean longitude in the orbit," is in reality the sum of two angles lying in different planes, viz., the longitude of the node + the angle between the radii from the Sun to the node and to an imaginary body starting from perihelion at the same epoch as segment A of the meteors, and thenceforward moving uniformly in a circular orbit round the Sun in the same plane and with the same periodic time as the meteors. So again π, the so-called "longitude of perihelion," is the sum of two angles, viz., the longitude of the node measured along the ecliptic, + the angle from the node to the perihelion measured in the plane of the orbit. The second angle in each

case, that in the plane of the orbit, is measured in the direction of the positive motion.

The perihelion distance in Adams' orbit, of which the elements are in the first column of the table, and which was the osculating ellipse on 1866, November 13, is 0.9855; that of the osculating ellipse on 1900, January 27, of which the elements are in the last column, is 0.97296. There is a corresponding difference in the distances of the node from the Sun, a difference which would be enough to carry segment A of the meteoric stream inside the Earth's orbit without intersecting it when it passes the Earth's orbit on 1900, January 27, unless the depth of the stream towards the Sun is greater than its width at right angles to that direction—a width which from observation has been estimated to be about 100,000 miles. We have, however, satisfied ourselves, from the dynamical conditions which must have prevailed when the Leonids joined the solar system, that the depth of the stream is much greater than its width.

The longitude of the node at the epoch 1900, January 27, would be 52° 25′, if computed in the way which has been hitherto usual, by applying to the longitude at the time of the shower of 1866 the average apparent shift of the node as determined from observation by Professor Newton, viz., 102.6″ annually; whereas in the orbit of our table it is 53° 42′. It thus appears that the amount of this perturbation upon segment A of the stream has been more than three and a half times its average amount, and, doubtless, the perturbations in this revolution of the other elements have also been excessive as compared with their average amounts.

Thus, the mean distance of the meteors occupying segment A of the stream has been undergoing so much extension, that the meteors will at the end of the revolution find themselves with a periodic time longer by one third of a year—an amount of change which must largely affect their future history, unless this great perturbation is compensated by what happens elsewhere or at other times.

At the epoch 1899, November 15, the longitude of the node

will be 53° 41.7′, a position which the Earth will reach on 1899, November 15^{d} 18^{h}. It is probable, therefore, that the middle of the shower of the present year (1899) will occur nearly at this time, since segment A in the stream, for which our calculations have been made, is situated in the stream less than three months' journey of the meteors behind the segment which the Earth will encounter next November, and which we may call segment B. This conclusion, however, rests on two assumptions: (1) that segments A and B were, in 1866, moving in orbits that did not much differ; (2) that the perturbations which segments A and B have since suffered have not much differed. Both assumptions are probable, but unfortunately neither is certain; so that the prediction can only be offered with reservation. If the shower occurs at the time anticipated, it will be visible from both Europe and America.

THE WAVE-LENGTH OF $H\delta$ AND THE APPEARANCE OF THE SOLAR SPECTRUM NEAR THE HYDROGEN LINES.

By LEWIS E. JEWELL.

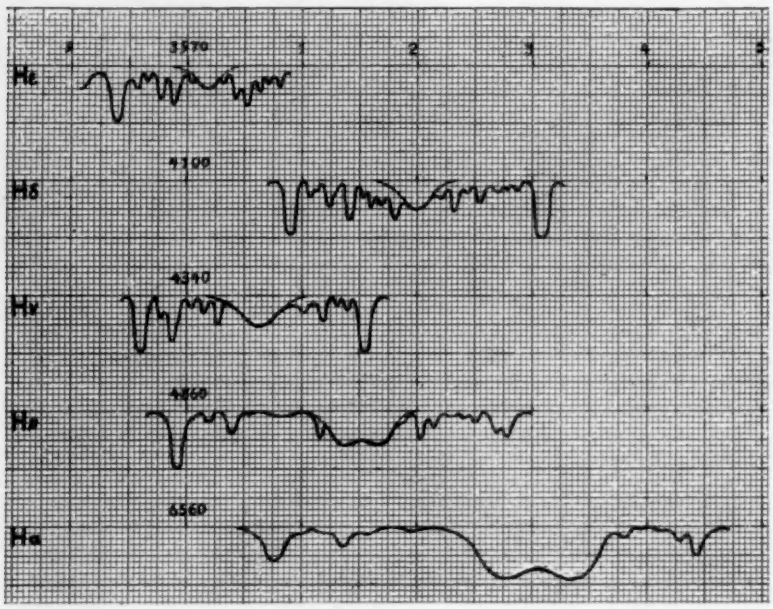
LAST summer Mr. J. Evershed, in working up his measurements of the hydrogen lines upon some remarkably fine plates of the "flash-spectrum," obtained at the time of the last solar eclipse in India, came to the conclusion that there was an error of about one tenth of an Ångström unit in the wave-length of $H\delta$ as given in Rowland's Tables.

More recently Mr. W. H. Wright and Professor W. W. Campbell, from their measurements of the hydrogen lines in the spectrum of *o* Ceti, have come to the same conclusion. Being responsible for the measurements and calculations of wave-length in Rowland's *Table of Solar Spectrum Wave-lengths*, and being also quite familiar with the appearance of $H\delta$ in the solar spectrum, I have thought it well to state the facts in the case as they appear to me.

The wave-length of $H\delta$ as given in the table is not a misprint, nor is there any error of observation or calculation, as has been ascertained by carefully going over the work again. However, there is some uncertainty as to the proper interpretation to be placed upon the appearance of the spectrum at this point. This region of the spectrum was measured upon a remarkably fine plate taken by Professor Rowland, and the table gives the appearance of the spectrum upon that plate according to my judgment, except that a very few *exceptionally* faint lines were not measured. A reëxamination of both this and a number of other plates confirms the original interpretation.

In the accompanying diagrams are given the intensity curves as determined from photographic plates where possible, which represent the appearance of the spectrum in the immediate vicinity of the five hydrogen lines in the visible portion of the

solar spectrum. The dotted lines show what the intensity curve of the hydrogen lines probably would be if free from the presence of other lines. The curves given show how difficult the interference of other lines renders the interpretation of phenomena in some cases. Even in the case of the hydrogen line C (*Ha*) there are some small water-vapor lines (not seen upon any plates which I have examined) which cross the broad nebulous hydrogen



line when the air is exceptionally moist and the Sun low. *Hγ* is freer from other lines than almost any of the other hydrogen lines, but there are indications that a small line due to some other element occurs at the center of this line. *Hδ* is the worst of all.

An examination of the intensity curves shows that there is a gradual reduction in the size and strength of the hydrogen lines of the solar spectrum as the violet end is approached; there is, however, a possible relative increase of the faint haze at the edges of lines. In the case of *Ha* and *Hβ* there is a faint reversal which is slightly displaced from the center of the line;

but in the case of $H\gamma$ there is no trace of a reversal. $H\epsilon$ is probably too faint and small to show a reversal even if one existed.

Upon Rowland's Map of the Solar Spectrum there is a shading towards the violet from the position assigned to $H\delta$ in Rowland's Table, in consequence of which $H\delta$ appears both much stronger and broader than $H\gamma$. The same appearance is shown upon plates where the definition is not of the very best quality, but upon the very finest plates where other lines are the sharpest and clearest, the indications are that this shading is due almost altogether to other faint lines and not to hydrogen. If it is due principally to hydrogen, then $H\delta$ is reversed in the solar spectrum and considerably stronger and broader than $H\gamma$, and the wave-length of the center of the line may be 4101.900 or slightly less; but, if $H\delta$ is not reversed in the solar spectrum and in intensity lies between $H\gamma$ and $H\epsilon$, then its wave-length is not less than 4102.000.

The relative intensity of the hydrogen lines in the chromospheric spectrum should show which conclusion is correct. It is not impossible that $H\delta$ may be both reversed and stronger than $H\gamma$, but the appearances upon the best solar spectrum plates are certainly against the supposition.

It may be well to mention in this connection that in the absorption spectrum of oxygen in the Earth's atmosphere, there is both an irregularity in the variation of intensity of the lines constituting the series in the tails of the oxygen bands, and at the same place there is a deviation from the positions of the lines as assigned by mathematical theory; possibly the same thing may be true with the hydrogen series. Both $H\delta$ and $H\epsilon$ as given by Balmer's formula differ from the positions in the solar spectrum given in the Table; and the difference is in opposite directions in the two cases.

The value of the wave-length for $H\delta$ as found for hydrogen gas in vacuum-tubes agrees with that deduced from Balmer's formula.

JOHNS HOPKINS UNIVERSITY.
March 6, 1899.

A DETERMINATION OF THE WAVE-LENGTHS OF THE PRINCIPAL LINES IN THE SPECTRUM OF GALLIUM, SHOWING THEIR IDENTITY WITH TWO LINES IN THE SOLAR SPECTRUM.

By W. N. HARTLEY and HUGH RAMAGE.

IT having been shown by us [1] in the examination of a number of minerals, such as feldspar, mica, basalt, pumice from Krakatoa, volcanic dust from New Zealand, iron ores, aluminous minerals, and of meteoric iron and meteoric dust, that gallium is a common constituent, present only in small proportion, it seemed of interest to determine whether traces of this element are to be found in the solar spectrum.

In order to test this matter by a more accurate investigation than is possible with ordinary instruments, we have been very glad to avail ourselves of the very kind offer of assistance made by Dr. W. E. Adeney, Curator of the Royal University of Ireland. He has afforded us the means of photographing spectra with the fine Rowland concave grating of twenty-one and a half feet radius which has been mounted in the Physical Laboratory of the University. The instrument was adjusted so that we could photograph on one plate, 19½ inches long, the region, in the second order, between wave-lengths 3990 and 4500. Cadet "lightning plates" were used, and they were developed with hydroquinone.

We were under the necessity of obtaining a specimen of pure iron for the purpose of obtaining a spectrum of this metal perfectly free from gallium, manganese, and one or two other elements, such as chromium, with which it is usually associated. For this purpose we made use of the iron in a pulverulent form, which is separated from potassium ferrocyanide when this sub-

[1] *Proc. Royal Society*, **60**, 35, and 393. *Trans. Chemical Society*, 1897, pp. 533, and 547. *Journal Iron and Steel Institute*, 1897, No. 2, p. 182. *Scientific Proc. Roy. Dub. Soc.*, **8** (N. S.), Part 6, No. 68.

stance is fused with potassium carbonate, and the black powder is separated from the potassium cyanide by solution in water or alcohol, and afterwards washed and dried. We believe this to be the purest form of iron which has yet been made.

To use it in the arc, we are obliged to ram it into carbon tubes, by which treatment it is unfortunately contaminated with carbon, but we have not found other impurities introduced. We have also used the iron residue obtained by the simple ignition of potassium ferrocyanide, the carbon of which must be very pure. For oxyhydrogen flame spectra, it is rolled up in ashless filter-papers and burnt in the flame.

From our knowledge of the spectrum of gallium and of the proportion present in the minerals containing it, we concluded that it would probably be useless attempting to find any lines in the solar spectrum other than the two well-known lines of wave-length about 4172 and 4033. We found that the less refrangible of these lines is nearly coincident with an iron line in the arc spectrum of iron and in the solar spectrum, and that the second and more refrangible line is nearly coincident with an iron-manganese line in the solar spectrum. In a case of this kind where the lines are very feeble and very closely adjacent to others, mere coincidence observed by photographing metallic spectra along with that of the Sun is not so satisfactory as actually determining the wave-lengths by measurements. The following is a list of the photographs taken with the Rowland grating :

Plate I. (1) Solar spectrum.
(2) Blast furnace iron containing $\frac{1}{30000}$ of its weight of gallium, an arc spectrum.

These spectra cannot be considered as showing absolute coincidences with the gallium lines. The arc spectrum contains a very large number of lines belonging to iron, but those of gallium are not distinctly visible, because the iron lines lie over them.

Plate II. (1) Spark spectrum taken from a solution of gallium chloride between platinum electrodes. Exposure 15 minutes.
(2) Solar spectrum showing where coincidences might be looked for.

This photograph gives the relative intensities of the two lines, the less refrangible being the stronger.

Plate III. A second plate with the same two spectra, a band of the solar spectrum running through the middle of the spectrum, and a second one touching the edges of the lines.

Plate IV. (1) An arc spectrum of pure iron, the metal being prepared from potassium ferrocyanide, by fusion with potassium carbonate.

(2) The same.

(3) The same, with a larger proportion of the residue obtained on ignition of gallium ferrocyanide.

(4) Similar to (3), but with a smaller proportion of gallium.

(5) Solar spectrum photographed on the succeeding day, the Sun at the time being too low to show possible coincidences.

In (3) and (4) the gallium lines are beautifully reversed; but in (4) the lines are broad and the reversals much less marked. The reversed lines of gallium are clearly seen to correspond with reversals in the solar spectrum; but the reversals may probably be those of iron lines very closely adjacent to those of gallium.

Plate V. (1) Arc spectrum of pure iron from ferrocyanide, with the addition of a gallium compound, on the middle portion only. The solar spectrum is taken with the middle portion cut out.

(2) Arc spectrum of a small quantity of a gallium compound and a small quantity of the iron also, with the solar spectrum as in (1).

In the oxyhydrogen flame, arc and spark spectra of substances both poor and rich in gallium, the line 4172 is always stronger than 4033.

By measurements of the iron lines and the gallium lines in arc spectra of materials containing different proportions of the two metals, the wave-lengths of the two gallium lines were determined by interpolation from the iron lines. The wave-lengths of the latter used were those determined by Rowland in the solar spectrum. By this method, the wave-lengths of the reversed gallium lines are found to be 4172.214 and 4033.125. These numbers are higher than those obtained by Lecoq de Boisbaudran in the spark, and higher also than our measurements of the lines in the oxyhydrogen spectra photographed with very small

dispersion, namely, 4171.6 and 4032.7; but they have been verified to the second decimal place by different measurements.

In Rowland's Table of Solar Spectrum Wave-Lengths, published in the ASTROPHYSICAL JOURNAL, Vol. I, pp. 139 and 225, there are two lines corresponding to these; but, to judge of the probability of these lines belonging to the element gallium, it is necessary to consider their relative intensities. Rowland measures the intensities of the solar lines over a wide scale, extending from 1, a line just clearly visible on the map, to 1000, for the H and K lines; and it is remarked that this is hardly enough for the enormous differences in intensity. Below 1, the lines in the order of faintness, proceed from 0 to 0000, indicating lines more and more difficult to see. The lines in his table which lie near to the two measured lines in the arc-spectrum of gallium are the following:

SOLAR LINES.[1]

λ	Intensity
4171.854, Cr, La, Mn, Ni, Fe,	3
4172.066, Ti, Fe,	2
4172.211, Al (?),	1
4172.296, Fe,	2
λ	
4032.610, Fe,	2
4032.789, Fe,	4
4032.985,	000
4033.112,	00
4033.224, Fe, Mn, S.,	7d

We find that the lines in the solar spectrum most nearly coincident with the gallium lines, according to our determinations of the wave-lengths, are the following:

Solar lines	Intensity	Gallium lines	Intensity
4172.211, Al,[2]	1	4172.214,	1
4033.112,	00	4033.125,	00

We believe these numbers to be quite accurate to the second decimal place. Our micrometer measures to the ten-thousandth of an inch, and an error of this magnitude makes a difference of

[1] This JOURNAL, 1, 1895.

[2] Corrected by Rowland, and definitely assigned to aluminium. See this JOURNAL, December 1897.

0.0033 in the wave-length. We used lines in the arc spectrum as fiducial lines, which correspond with the following solar lines :

SOLAR LINES.[1]

4191.595, Fe, 6	4044.766, Fe, 3
4187.204, Fe, 6	4040.792, Fe, 3
4175.806, Fe, 5	4034.644, Mn–Fe, 6
4171.068, Fe, 4	4032.789, Fe, 4
4143.572, Fe, 4	4032.117, Fe, 2

The wave-lengths of the two gallium lines determined from these are as follows :

Reversed lines, Plate IV, - - -	4172.214 and 4033.125
Lines in Plate V, - - - -	4172.214 and 4033.120

With the stronger line the numbers vary between 4172.210 and 4172.216, and with the weaker line between 4033.117 and 4033.128.

The relative intensities of the two gallium lines are the same in the oxyhydrogen flame, the arc (bright and reversed lines), and spark spectra ; and they are fairly represented on Rowland's scale by 1 and 00.

We consider the wave-lengths determined from the reversed lines to be more accurate than those determined from the bright lines in Plate V. In the latter the gallium lines and the closely adjacent iron lines overlap. We therefore adopt 4172.214 and 4033.125 as the wave-lengths of two lines in the spectrum of gallium which have been observed in various substances examined by us.

There are two lines, 4172.296, Fe, and 4033.224, Fe–Mn, which are so closely adjacent that we have not been able to distinctly separate them from the gallium lines, even when working on spectra of the second order, though the ends of the two lines can be observed with the microscope quite distinctly. By working in a clearer atmosphere, with a higher order of spectrum and a narrower slit, it may be possible to distinctly separate two Fraunhofer lines of these wave-lengths.

[1] This JOURNAL, **1**, February and March 1895, and **6**, December 1897.

The evidence that gallium is contained in the Sun is of the following character:

1. This element, in minute proportions, is extraordinarily widely distributed in the crust of the Earth, in felspar, mica, basalt, iron ores, and aluminous minerals generally. It is also commonly found, as we have ascertained, in pumice and volcanic dust from New Zealand and Krakatoa; thus proving its presence in the interior of the Earth.

2. Gallium is a common constituent of iron meteorites, associated with nickel and cobalt.[1]

3. The lines of gallium, both in the arc and spark spectra of a solution of gallium chloride, show that the less refrangible is the stronger line, and that their relative intensities are represented by 1 and 00 on Rowland's scale.

4. In the arc spectrum of gallium these two lines are very easily reversed.

5. The wave-lengths of the gallium lines, 4172.214 and 4033.125, correspond with two lines in the solar spectrum, one of which has been assigned to aluminium by Rowland, the wave-lengths of which are 4172.211 and 4033.112.

As owing to the chemical properties of gallium oxide — its separation from alumina and other sesquioxide bases is extremely difficult, and requires a very special and peculiar treatment — we should expect to find that aluminous compounds, from whatever source, and aluminium would furnish the gallium lines, which is exactly in accordance with our experience. There can be no doubt, from the relative intensities of the lines, from their wave-lengths, from the association of gallium with aluminium and with iron, that the solar lines 4172.211 and 4033.112 have their origin from gallium contained in the Sun, which is present in small proportion when compared with iron, and that the solar spectrum, in so far as the proportion of gallium to iron is concerned, may be considered to be fairly imitated by the arc spectrum of blast-furnace iron containing $\frac{1}{30000}$ of its weight of gallium, since, if the more volatile metal were in any consid-

[1] *Scientific Proc. Roy. Dub. Soc.*, **8** (N. S.), Part 6, p. 705.

erable proportion, the gallium lines would broaden and overlap those of iron with wave-lengths 4172.296 and 4033.224.

This research brings to light the fact that, where coincidences are few in number, the mere coincidences of lines in the spectra of terrestrial matter with reversed lines in the solar spectrum is not equivalent to a proof of the existence of the elements in the Sun or other heavenly bodies, even when the most powerful instruments are employed for resolving the lines. Professor Rowland's tables of solar spectrum wave-lengths show not only how nearly lines of different elements may coincide, but how there are some actual coincidences, as for instance of nickel with iron lines. Lines may also overlap. Generally speaking, two lines of the same wave-length, belonging to different elements, differ in this respect, that one is strong and the other weak, or perhaps not so strong.

Examples are familiar to us and may be cited. For instance, two lines of rubidium are very frequently obscured by two of iron; the strong line of rubidium corresponds with the weak line of iron, and *vice versa*.[1] If therefore the two lines appear of the same intensity, we know that rubidium is present; and if the order of the intensity is the reverse of that of the iron lines, we know then that the proportion of rubidium is larger than in the former case. Of course, the presence of iron is determined by other lines than these two which coincide with the rubidium lines. The greater mass of a substance may have the effect of obscuring or extinguishing some of the lines in the spectrum of another element less easily volatilized. On the other hand, the greater mass of a less easily vaporized substance may also obscure the lines of one more volatile which are in close proximity.

In conclusion, we tender our sincere thanks to Dr. Adeney for the aid so cordially given us in obtaining photographs from which these measurements have been made.

[1] Wave-lengths of the rubidium lines, 4215.72 and 4201.98; wave-lengths of the iron lines, 4216.28 and 4202.15.

A SPECTROGRAPHIC ANALYSIS OF IRON METEORITES, SIDEROLITES, AND METEORIC STONES.[1]

By W. N. HARTLEY and HUGH RAMAGE.

IN a paper just published in the *Transactions* of the Chemical Society, **51**, 533, on "The Wide Dissemination of Some of the Rarer Elements, and the Mode of their Association in Common Ores and Minerals," we have shown that out of ninety-one iron ores belonging to the metallurgical collection in the Royal College of Science, thirty-five contain the extremely rare metal gallium, and most of them contain constituents of an unusual character not hitherto known or suspected to be contained therein. For instance, rubidium appears to be very commonly present, while the magnetites, from whatever part of the world, invariably contain gallium, but no indium; the siderites all contain indium, but no gallium. We have, therefore, considered it desirable that meteorites should be examined, and accordingly a selection of specimens was made for the purpose. They are classified into meteoric irons, siderolites, and meteorites, or meteoric stones, and on the following pages their composition is shown, with the collections from which they were obtained.

In the paper mentioned we have given a list of those elements capable of being detected by our method of examination. Those which are not volatilized in the oxyhydrogen flame are silicon, titanium, vanadium, tungsten, platinum, etc., and these have not been sought for.

The range of spectrum examined is extensive, and lies between wave-lengths 6000 and 3200, and lines capable of being photographed were carefully observed. It will be noted that in the tabulated statement (on page 223), after the symbol of the element, an index number from 1 to 9 shows the relative

[1] *Scientific Proceedings of the Royal Dublin Society*, Vol. VIII (N. S.), Part 6, No. 68.

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strength of the lines, the figure 1 indicating the weakest, and 9 the strongest appearance of the same lines in the several spectra. In the case of the principal constituents of the meteorites this is unnecessary, as lists of the lines measured are given, but where only two or three lines are visible, the substances being in minute proportions, the index figures serve a very useful purpose. Symbols in italics indicate traces only. The tabulated statement clearly shows the elements which are present, with variations in the composition of the different specimens.

The lines were identified by measurements made on the photographs, and wave-lengths were obtained from curves based on Rowland's Standard, the particular wave-lengths quoted being those of Kayser and Runge. Lists of the lines of iron, nickel, cobalt, sodium, potassium, rubidium, gallium, copper, silver, and lead are given. A calcium line was recorded in all specimens, and the *bands of magnesia* in all the stony meteorites.

We were at first inclined to doubt whether calcium, sodium, and potassium were really constituents of the meteoric irons, but the lines of the alkali metals, which are very weak, were proved to belong to the metallic iron by burning it without having recourse to a support of any kind, and thus the spectrum observed was the same as that obtained by burning filings of the metal on supports. A portion of the file used upon the metal was also burned, and this showed a composition differing from that of the meteoric irons, since it contained manganese, but no nickel, cobalt, or gallium.

THE COMPOSITION OF METEORIC IRONS, SIDEROLITES, AND METEORITES.

Country and date	Calcium	Iron	Nickel	Cobalt	Chromium	Gallium	Lead	Manganese	Silver	Copper	Sodium	Potassium	Rubidium
METEORIC IRONS.													
1. Virginia (1) (*a*)	Ca	Fe	Ni	.....	.....	Ga 2	.....	.....	Ag 1	.....	*Na*	.....	Rb
2. Virginia (2). Royal College of Science Museum, Dublin (*a*)	Ca	Fe	Ni	*Co*	.....	Ga 1	Pb 4	.....	Ag 1	Cu 2	*Na*	*K*	Rb
3. Iron, with troilite, Arva, Hungary (*a*)	Ca	Fe	Ni	*Co*	.....	Ga 3	Pb 6	*Mn*	Ag 1	Cu 4	Na	*K*	Rb
4. Coahuila, Mexico. Fell 1860 (*b*)	Ca	Fe	Ni	*Co*	.....	Ga 1	Pb 2	.....	Ag 1	Cu 1	Na	*K*	Rb
5. Tolucca, Mexico. Found 1784 (*b*)	Ca	Fe	Ni	Co	.....	Ga 3	Pb 1	.....	Ag 1	Cu 2	Na	*K*	Rb
6. Cañon Diablo, Yarapia county, Arizona. Found 1891 (*b*)	Ca	Fe	Ni	Co	.....	Ga 2	Pb 4	*Mn*	Ag 1	Cu 4	Na	*K*	Rb
7. Thunda, Windorah, Queensland, Fell December 1886 (*b*)	Ca	Fe	Ni	Co	.....	*Ga* 1	Pb 2	.....	Ag 1	Cu 2	Na	*K*	Rb
SIDEROLITES.													
8. Atacama, Chili. Found 1858 (*a*)	Ca	Fe	Ni	*Co*	.....	Ga 1	Pb 4	?	Ag 3	Cu 3	Na	*K*	Rb
9. Estherville, Emmet county, Iowa. Fell May 10, 1879 (*b*)	Ca	Fe	Ni	*Co*	*Cr* 1	.....	Pb 2	?	Ag 1	Cu 2	Na	*K*	Rb
10. Imilac, Atacama, Chili. Found 1829 (*b*)	Ca	Fe	Ni	Co	.....	.....	Pb 1	?	.....	Cu 2	Na	*K*	Rb
METEORITES.													Magnesium
11. Alfianella, Lombardy. February 16, 1883 (*a*)	Ca	Fe	Ni	.....	Cr 3	*Ga*	Pb 1	*Mn*	Ag 1	Cu 2	Na	*K*	MgO
12. Pultusk, January 30, 1868 (*a*)	Ca	Fe	Ni	.....	Cr 2	.....	Pb 1	*Mn*	Ag 2	Cu 2	Na	*K*	MgO
13. Mocs, Transylvania (*a*)	Ca	Fe	Ni	.....	Cr 3	.....	Pb 1	*Mn*	Ag 1	Cu 2	Na	*K*	MgO

(*a*) Specimens from the collection in the Royal College of Science, Dublin.

(*b*) Specimens purchased from Messrs. James R. Gregory & Co., 1 Kelso Place, Kensington, London.

It is difficult to determine whether rubidium is present on account of the proximity of two iron lines, but the more refrangible is the stronger of the rubidium lines; it is exactly the reverse with the iron lines. Their wave-lengths are: rubidium, 4215.72, 4201.98; iron, 4216.28, 4202.15.

LIST OF IRON LINES MEASURED AND IDENTIFIED IN THE SPECTRA OF METEORIC IRONS; WAVE-LENGTHS ACCORDING TO KAYSER AND RUNGE'S MEASUREMENTS.

	λ		λ		λ		λ	
	5371.60		4045.90		3813.12		3682.35	
	5330.90[1]		4005.33		3799.68		3680.03	
E	5269.65		3969.30		3798.65		3647.99	
	5169.09		3930.37		3795.13		3631.62	
	5110.50		3928.05		3788.01		3618.92	
	4482.35		3923.00		3765.66		3608.99	
	4461.75		3920.36		3763.90		3587.10	
	4427.44		3906.58		3758.36		3586.24	
	4415.27		3899.80		3749.61	N	3581.32	
	4404.88		3895.75		3748.39		3570.23	
	4383.70		3887.17		3745.67		3565.50	
	4376.04		3878.82		3743.45		3558.62	
	4325.92		3872.61		3737.27		3526.50	Ni here
G	4307.96		3867.33		3735.00		3521.56	
	4294.26		3860.03		3732.54		3513.91	
	4271.30		3856.49	M	3727.78		3497.92	
	4250.93		3850.11		3727.13		3490.65	
	4216.28		3840.58		3722.59		3476.75	
	4202.15		3834.37		3720.07		3475.52	
	4143.96		3826.04		3709.37		3465.95	
	4132.15		3824.58		3708.03	O	3441.07	
	4071.79	L	3820.56		3707.18		3440.69	
	4063.63		3815.97		3687.77			

LINES OF NICKEL IN METEORIC IRONS; WAVE-LENGTHS ACCORDING TO LIVEING AND DEWAR.

λ	λ	λ	λ	
3857.8	3565.7	3461.1	3390.4	
3806.6	3547.5	3457.9	3380.0	
3783.6	3523.9	3452.3	3373.3	
3775.0	3519.1	3445.7	3371.3	
3618.8	3514.4	3436.7	3368.9	
3612.1	3509.7	3433.0	3365.4	Appears
3609.8	3492.3	3423.1	3365.1	as one line
3597.0	3483.1	3413.8	3319.7	
3571.2	3471.9	3412.9	3315.1	

[1] Not identified on Kayser and Runge's map; possibly it is 5330.15, but in our spectra there is a diffuse band of rays hereabouts, and apparently several feeble lines or edges of bands, which are difficult to measure.

LINES OF COBALT IN METEORIC IRONS; LIVEING AND DEWAR'S WAVE-LENGTHS.

λ	λ	
4119.4	3533.6	
3995.4	3529.9	Very strong
3575.7	3529.0	
3575.3	3526.9	Possibly two lines
3569.7	3502.2	

LINES OF THE ALKALI METALS IN METEORIC IRONS; KAYSER AND RUNGE'S MEASUREMENTS.

Sodium	Potassium	Rubidium
λ	λ	λ
5896.16	4047.36	4201.98
5890.19	4044.29	4215.72
3303.07		

LINES OF OTHER METALS IN METEORIC IRONS.

Gallium	Copper	Silver	Lead
λ	λ	λ	λ
4171.8	3274.09	3382.98	4057.97[1]
4032.7	3247.65	3280.80	3683.60
			3639.70

A calcium line was observed, wave-length 4226.91. This was believed to be caused by dust. There is an iron line at 4227.60 (K. and R.), but it does not appear in flame spectra. In the meteoric irons from Arva, Hungary, in which there is troilite, and in the specimen from Cañon Diablo, Arizona, there is a trace of manganese. The lines just visible are those with wave-lengths 4033 and 4030 (Liveing and Dewar's numbers are 4032.0 and 4029.5).

The nickel and iron lines are strong in all the specimens. The alkali metals are weak, potassium being found in traces only. These spectra disclose a marked difference between meteoric iron and telluric metal, not only in the presence of a large proportion of nickel in the former, but in the absence of manganese, an element which is invariably contained in the latter.

[1] This line is shown as 4357 in Plate 7, *Phil. Trans.*, **185**, 1894, by an error of the engraver.

The presence of gallium in variable proportions in the iron meteorites is remarkable. The only one in which its occurrence was at first doubtful was that from Thunda, Windora, Queensland. Cobalt occurs in all specimens except in one from Virginia, but it does not appear in the meteoric stones.

The meteoric stones all contain chromium in variable proportions and manganese in traces only.

The meteorite from Atacama consists of a honeycombed mass of iron, the spaces being filled with a yellow crystalline mineral (olivine ?), which was examined separately from the iron, and found to contain the following constituents, the bases being separated from the silica.

COMPOSITION OF THE NON-METALLIC PORTION OF THE IRON METEORITE FROM ATACAMA.

Alkali and alkaline earth metals.—Sodium, potassium, magnesium, calcium, and a trace of strontium present as oxides or silicates.

Heavy metals.—Iron, nickel, chromium, copper, silver, lead, and a trace of manganese as oxides or silicates.

Professor Norman Lockyer has examined the photographic arc spectra of iron meteorites (*Phil. Trans.*, **185**, 1023), using as poles pieces of the meteoric irons from the British Museum, known as the Nejed and Obernkirchen meteorites. The range of spectrum was from about 5892 (D) to 3933 (K). In addition to iron the following substances were declared to be present: Manganese, cobalt, nickel, chromium, titanium, copper, barium, calcium, sodium, and potassium. Others were said to be probably present, namely, strontium, lead, lithium, molybdenum, vanadium, didymium, uranium, tungsten, yttrium, osmium, and aluminium.

The general conclusions arrived at were that the two meteorites agreed very closely in composition; that there was a very considerable similarity between the spectra of the meteorites and that of the Sun, the lines having the same relative intensity as those in the solar spectrum. The presence of copper was

supposed to be probably due to copper wire being used to bind the pieces of iron to the poles of the arc lamp, as neither flame nor spark spectra confirmed the presence of copper. It may here be remarked, however, that the most prominent lines in the spectrum of copper lie in a region far beyond K in the ultra-violet, and were therefore not within Lockyer's range of observation, when produced either by arc, spark, or flame. There were forty-three lines for which no origin was suggested, twenty-nine being apparently coincident with lines in Kayser and Runge's iron spectrum. It was shown that the chief chemical differences between the two meteorites was a preponderance of calcium in the Nejed meteorite, and of nickel, barium, and strontium in that from Obernkirchen. A line at 4171.2 is described as "unknown," and one at 4031.4 is doubtfully ascribed to iron. The former is certainly, and the latter probably, a gallium line, wave-lengths 4171.8 and 4032.7.

The substances which yield spectra in the oxyhydrogen blowpipe, capable of being photographed, are those which are easily volatilized at a temperature of about 1800 C., as one of us has already shown ("Flame Spectra at High Temperatures, Part I, Oxyhydrogen Blowpipe Spectra," *Phil. Trans.*, **185**, 161, 1894), and they form a very large proportion of the metallic elements and their compounds. When examined by this method over a wide range in the ultra-violet, most substances yield characteristic spectra.

CONCLUSIONS.

1. The composition of different metallic ores is very similar, though the proportions of the constituents differ to some extent.

2. We find that copper, lead, and silver are common constituents of meteoric irons, and that they occur in variable proportions. We have already shown that this is the case with iron ores of different varieties, and different kinds of manufactured irons.

3. Gallium is a constituent in varying proportions of all meteoric irons, but not of all meteorites. It occurs in one of the siderolites we have examined.

4. Sodium, potassium, and rubidium are constituents of meteoric irons, but only in minute proportions.

5. Chromium and manganese are found in meteoric stones, but not in the irons, though very minute traces of manganese have been detected in two of our specimens.

6. Nickel is found as a principal constituent in all meteorites, meteoric irons, and siderolites. Cobalt occurs in the two latter varieties only.

The chief points of difference between telluric and meteoric iron is the absence of nickel and cobalt in any considerable proportion from the former, and the presence of manganese; while meteoric irons contain nickel and cobalt as notable constituents, and, except in minute traces, manganese is absent.

NOTES ON THE PAPERS OF HARTLEY AND RAMAGE CONCERNING THE SPECTRUM OF GALLIUM AND THE SPECTRA OF METEORITES.[1]

By LEWIS E. JEWELL.

THE two papers upon the above subjects have recently appeared and contain matter of much interest. When the plates of the metallic spectra were taken in the laboratory at Johns Hopkins University, Professor Rowland could secure no specimen of gallium or any of its salts. In the light of the researches of Professors Hartley and Ramage I have examined all of the plates likely to contain gallium.

The aluminium plates show the two gallium lines fairly strong, and the lead plates somewhat less so. The silver plates show them faintly, as also do some of the plates of iron. Of the meteorites, the plates of the spectrum of the Bendigo meteorite show them faintly, while the New Concord meteorite shows no indications.

One plate each of aluminium and lead shows the gallium lines quite strong, and the coincidence of the two lines at 4033.112 and 4172.211 with solar lines seems to be exact.

Rowland's revised tables of wave-lengths for the regions given by Hartley and Ramage will be as follows:

4032.418	-Sr	00	4171.597	-,Zr	00
4032.610	Fe?-V?	2	4171.720	Cr	00
4032.789	Fe	4	4171.854	C, Fe	2
4032.985		000	4172.066	Ti,Fe	2
4033.112	Ga	00	4172.211	Ga	1
4033.224 s	Mn	*8d?	4172.296	Fe-Ce	2

*Apparent duplicity caused by reversal in Sun.

METEORITIC SPECTRA.

The spectra of several meteorites have been photographed in the Johns Hopkins University. Fairly complete spectra have been obtained of the New Concord, Ohio, and the Bendigo meteorites, and one plate each of the Tolucca, Mexico, and Fayette county, Texas, meteorites.

[1] See pp. 214 and 221.

The New Concord meteorite gives quite a remarkable spectrum. The substances whose presence is indicated are as follows, in the order of their importance as determined by the lines of their spectra present: Mg, Na, Ca, Mn, Fe, Ni, Si, Al, Cr, Ba, Ti, Co, V, Sr, and K. There may be a trace of Rb, and the violet copper lines show somewhat stronger than they would from carbon poles alone, but the green copper lines are not seen upon the plates. The spectrum is chiefly remarkable for the large amounts of Mg, Na, Ca, and Mn present, although there is considerable Si, Al, Cr, and Ti also.

Notwithstanding the character of the spectrum and the fact that the chippings easily powdered up, the surface of the meteorite was harder than chilled steel, breaking up the cold chisels with which the chippings were secured. I am indebted for the chippings to the late Wm. C. Gurley, formerly director of the Marietta, Ohio, Observatory, the meteorite being in the museum of Marietta College. It was seen to fall at New Concord, Ohio, on May 1, 1860, at 12:30 P. M.

The Fayette county, Texas, meteorite has a similar spectrum, the one plate taken showing Mg, Ca, Fe, Cr, Fe, Ti, and Ni. It contains relatively more Ti and Cr, and somewhat less Mn, Ni, and Na.

The Bendigo meteorite is distinctly of the iron class, its spectrum showing Fe, Ni, Co, and Ca. Al, Rb, V, and Ga are probably present in minute quantities only, while the spectrum shows no traces of Mn, Cr, Mg, Ca, Na or Si. The copper lines in the green show distinctly, as well as the two strong lines in the ultra-violet.

The spectrum of the Tolucca meteorite shows the presence of Fe, Ni, Cr, and Mg. Other substances are not indicated upon the plate examined, except, perhaps, carbon. For some reason all of the metallic lines were very difficult to bring out in the spectrum of this meteorite. This was even more noticeable in visual observations than in the photograph of the arc.

JOHNS HOPKINS UNIVERSITY,
March 9, 1899.

APPLICATION OF SELLMEIER'S DYNAMICAL THEORY TO THE DARK LINES D_1, D_2 PRODUCED BY SODIUM-VAPOR.[1]

By LORD KELVIN.

1. FOR a perfectly definite mechanical representation of Sellmeier's theory, imagine for each molecule of sodium-vapor a spherical hollow in ether, lined with a thin rigid spherical shell, of mass equal to the mass of homogeneous ether which would fill the hollow. This rigid lining of the hollow we shall call the sheath of the molecule, or briefly the sheath. Within this put two rigid spherical shells, one inside the other, each movable and each repelled from the sheath with forces, or distribution of force, such that the center of each is attracted towards the center of the hollow with a force varying directly as the distance. These suppositions merely put two of Sellmeier's single-atom vibrators into one sheath.

2. Imagine now a vast number of these diatomic molecules, equal and similar in every respect, to be distributed homogeneously through all the ether which we have to consider as containing sodium-vapor. In the first place, let the density of the vapor be so small that the distance between nearest centers is great in comparison with the diameter of each molecule. And in the first place also, let us consider light whose wave-length is very large in comparson with the distance from center to center of nearest molecules. Subject to these conditions we have (Sellmeier's formula)

$$\left(\frac{v_e}{v_s}\right)^2 = 1 + \frac{m\tau^2}{\tau^2 - \kappa^2} + \frac{m_1\tau^2}{\tau^2 - \kappa_1^2}; \qquad (1)$$

where m, m_1 denote the ratios of the sums of the masses of one and the other of the movable shells of the diatomic molecules in any large volume of ether, to the mass of undisturbed ether

[1] *Phil. Mag.*, (5) **47**, March 1899.

filling the same volume; κ, κ_1 the periods of vibration of one and the other of the two movable shells of one molecule, on the supposition that the sheath is held fixed; v_e the velocity of light in pure undisturbed ether; v_s the velocity of light of period τ in the sodium-vapor.

3. For sodium-vapor, according to the measurements of Rowland and Bell,[1] published in 1887 and 1888 (probably the most accurate hitherto made), the periods of light corresponding to the exceedingly fine *dark* lines D_1, D_2 of the solar spectrum are .589618 and .589022 of a michron.[2] The mean of these is so nearly one thousand times their difference that we may take

$$\kappa = \frac{1}{2}(\kappa + \kappa_1)\left(1 - \frac{1}{2000}\right); \quad \kappa_1 = \frac{1}{2}(\kappa + \kappa_1)\left(1 + \frac{1}{2000}\right). \tag{2}$$

Hence if we put $$\tau = \frac{1}{2}(\kappa + \kappa_1)\left(1 + \frac{x}{1000}\right), \tag{3}$$

and if x be any numeric not exceeding 4 or 5 or 10, we have

$$\left(\frac{\kappa}{\tau}\right) \doteq 1 - \frac{1}{1000}(2x + 1); \quad \left(\frac{\kappa_1}{\tau}\right)^2 \doteq 1 - \frac{1}{1000}(2x - 1); \tag{4}$$

whence $$\frac{\tau^2}{\tau^2 - \kappa^2} \doteq \frac{1000}{2x + 1}; \quad \frac{\tau^2}{\tau^2 - \kappa_1^2} \doteq \frac{1000}{2x - 1}. \tag{5}$$

Using this in (1), and denoting by μ the refractive index from ether to an ideal sodium-vapor with only the two disturbing atoms m, m_1 we find

$$\left(\frac{v_e}{v_s}\right)^2 = \mu^2 = 1 + \frac{1000\, m}{2x + 1} + \frac{1000\, m_1}{2x - 1}. \tag{6}$$

4. When the period, and the corresponding value of x according to (3), is such as make μ^2 negative, the light cannot enter the sodium-vapor. When the period is such as to make μ^2 real, the proportion (according to Fresnel, and according to the most

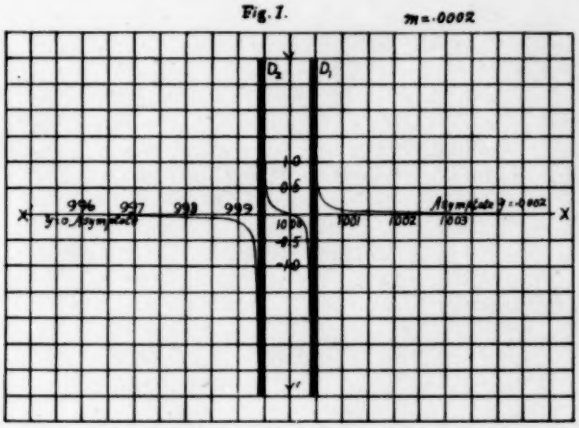
[1] ROWLAND, *Phil. Mag.*, (5) **23**, 265–282, 1887; Bell, *Phil. Mag.*, (5) **25**, 350–372, 1888.

[2] "Michron" is the name which I have given to a special unit of time such that the velocity of light is one mikrom of space per michron of time, the mikrom being one millionth of a meter. The best determinations of the velocity of light in undisturbed ether give 300,000 kilometers, or 3×10^{14} mikroms, per second. This makes the michron $\frac{1}{3} \times 10^{-14}$ of the second.

probable dynamics), of normally incident light which enters the vapor is

$$1 - \left(\frac{\mu - 1}{\mu + 1}\right)^2. \qquad (7)$$

5. Judging from the approximate equality in intensity of the bright lines D_1, D_2 of incandescent sodium-vapor; and from the approximately equal strengths of the very fine dark lines D_1, D_2 of the solar spectrum; and from the approximately equal strengths, or equal breadths, of the dark lines D_1, D_2 observed in the analysis of the light of an incandescent metal, or of the

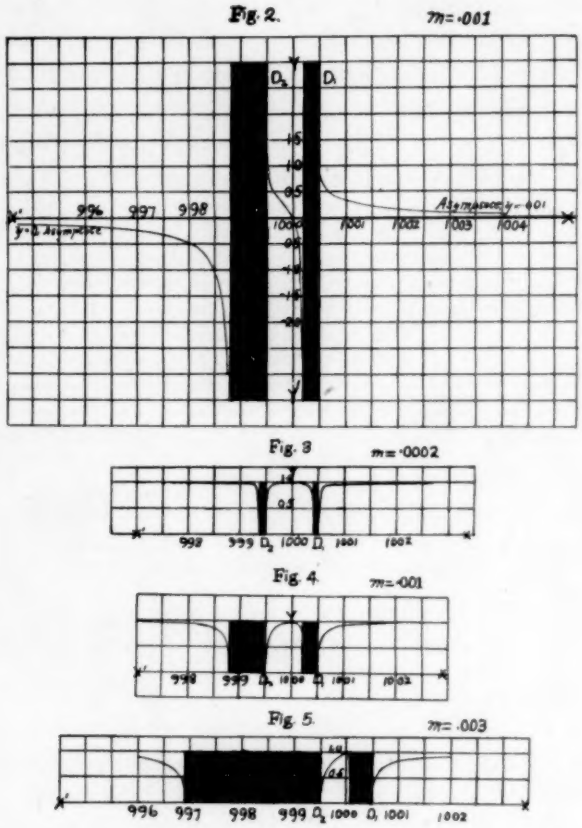


electric arc, seen through sodium-vapor of insufficient density to give much broadening of either line; we see that m and m_1 cannot be very different, and we have as yet no experimental knowledge to show that either is greater than the other. I have therefore assumed them equal in the calculations and numerical illustrations described below.

6. At the beginning of the present year I had the great pleasure to receive from Professor Henri Becquerel, inclosed with a letter of date December 31, 1898, two photographs of anomalous dispersion by prisms of sodium-vapor,[1] by which I was

[1] A description of Professor Becquerel's experiments and results will be found in *Comptes Rendus*, December 5, 1898, and January 16, 1899.

astonished and delighted to see not merely a beautiful and perfect demonstration of the "anomalous dispersion" towards infinity on each side of the zero of refractivity, but also an illustration of the characteristic nullity of absorption and finite breadth of

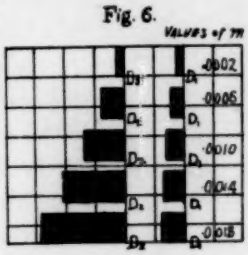


dark lines, originally shown in Sellmeier's formula[1] of 1872, and now, after twenty-seven years first actually seen. Each photograph showed dark spaces on the high sides of the D_1, D_2 lines, very narrow on one of the photographs; on the other much broader, and the one beside the D_2 line decidedly broader than the one beside the D_1 line; just as it should be according to Sell-

[1] SELLMEIER, *Pogg. Ann.*, **145**, 399, 520, 1872; **147**, 387, 525, 1872.

meier's formula, according to which also the density of the vapor in the prism must have been greater in the latter case than in the former. Guessing from the ratio of the breadths of the dark bands to the space between their D_1, D_2 borders, and from a slightly greater breadth of the one beside D_2, I judged that m must in this case have been not very different from .0002; and I calculated accordingly from (6), the accompanying graphical representation showing the value of $1 - \frac{1}{\mu}$, represented by y in Fig. 1. Fig. 2 represents similarly the value of $1 - \frac{1}{\mu}$ for $m = .001$, or density of vapor five times that in the case represented by Fig. 1. Figs. 3 and 4 represent the ratio of intensities of transmitted to normally incident light for the densities corresponding to Figs. 1 and 2; and Fig. 5 represents the ratio for the density corresponding to the value $m = .003$. The following table gives the breadths of the dark bands for densities of vapor corresponding to values of m from .0002 to fifteen times that value; and Fig. 6 represents graphically the breadths of the dark bands and their positions relatively to the bright lines D_1, D_2 for the first five values of m in the table.

Fig. 6.



Values of m	Breadths of bands	
	D_1	D_2
.0002	.09	.11
.0006	.217	.383
.0010	.293	.707
.0014	.340	1.060
.0018	.371	1.429
.0022	.392	1.808
.0026	.408	2.192
.0030	.419	2.581

7. According to Sellmeier's formula the light transmitted through a layer of sodium-vapor (or any transparent substance

to which the formula is applicable) is the same whatever be the thickness of the layer (provided of course that the thickness is at least several wave-lengths, and that the ordinary theory of the transmission of light through thin plates is taken into account when necessary). Thus the D_1, D_2 lines of the spectrum of solar light, which has traveled from the source through a hundred kilometers of sodium-vapor in the Sun's atmosphere, must be identical in breadth and penumbras with those seen in a laboratory experiment in the spectrum of light transmitted through half a centimeter or a few centimeters of sodium-vapor, of the same density as the densest part of the sodium-vapor in the portion of the solar atmosphere traversed by the light analyzed in any particular observation. The question of temperature cannot occur except in so far as the density of the vapor, and the clustering in groups of atoms, or non-clustering (mist or vapor of sodium), are concerned.

8. A grand inference from the experimental foundation of Stokes and Kirchhoff's original idea is that the periods of molecular vibration are the same to an exceedingly minute degree of accuracy through the great differences of range of vibration presented in the radiant molecules of an electric spark, electric arc, or flame, and in the molecules of a comparatively cool vapor or gas giving dark lines in the spectrum of light transmitted through it.

9. It is much to be desired that laboratory experiments be made, notwithstanding their extreme difficulty, to determine the density and pressure of sodium-vapor through a wide range of temperature, and the relation between density, pressure, and temperature of gaseous sodium.

THE PRESENT STATUS OF KIRCHHOFF'S LAW.[1]

By A. COTTON.

KIRCHHOFF's memoir[2] "On the Ratio between the Emissive and Absorptive Powers of Bodies for Heat and Light," appeared in 1859. Kirchhoff had then just made his experiments on the reversal of lines, which resulted in the explanation of the dark lines of the solar spectrum.

However, in this memoir these brilliant discoveries occupy little space. Kirchhoff presented them simply as an application of a general law establishing a relation between the absorption and the emission *of a given radiation.* A relation between the absorption and the emission of what is still called "radiant heat" had long been known, and De la Provostaye and Desains had verified it with great care. Kirchhoff reduced this relation to more definite form, showed that it must be true for each radiation considered by itself, and, adding a few words to the old statement, formulated an entirely new law.[3] At a given temperature the ratio between the emissive power and the absorptive power suitably defined, *and corresponding to a given radiation*, is the same for all bodies.

Since the appearance of Kirchhoff's memoir there have been numerous investigations on radiant heat and light. Thanks to the improvements of experimental methods, we are better acquainted at the present time with these thermal radiations, in the midst of which we live. Formerly the only means of studying the composition of thermal beams was by studying their absorption by screens (the same method which, in lack of a better one, is now used for the recently discovered rays).

At the present time the direct study of infra-red spectra has become much more simple. We can no longer give *without expla-*

[1] Translated from *Revue Générale des Sciences*, February 15, 1899.

[2] *Poggendorf's Annalen*, 1859.

[3] Here, as well as in the case of the reversal of lines, Kirchhoff had been anticipated. But the statement of the law had not yet been formulated with precision.

nation, as an experimental proof of Kirchhoff's law, the results of measurements of "radiant heat."

Further, the question of the origin of light, important even from a practical point of view, as it is related to the solution of the problem of economical illumination, has occupied the attention of many physicists. The phenomena of fluorescence, phosphorescence, etc., appear more and more important; under the name of *luminescence* (E. Wiedemann), are included the very numerous cases in which the nature and the intensity of the radiations emitted by a body are not determined by the temperature alone. Is the law of Kirchhoff general, and does it apply to all these cases? Or is it, on the contrary, insufficient to account for the phenomena of luminescence, and may these phenomena be thus distinguished?

This question and many others related to this subject have been the object of numerous researches, some of which are recent. Profiting by these investigations, I shall examine in a more general manner the present status of Kirchhoff's law.

Two distinct relations are almost invariably confused. This confusion, while permissible in Kirchhoff's time, is no longer so.

I shall study at the outset the *qualitative rule* which connects the absorption and emission *for a given body*, and consider how this rule should be expressed. The only conclusion that can be drawn from it is that if a body emits certain radiations, it absorbs them when they come to it from without. The phenomena of the reversal of spectral lines may be regarded as special cases of this very general rule.

I shall finally examine *Kirchhoff's law, properly so-called*. This law establishes a relation between different bodies, and completely defines the ratio $\frac{e}{a}$ between the emissive power and the absorptive power (suitably defined): this ratio is a known function of the temperature and of the wave-length, *a function which is the same for all bodies.*

This law, as we shall see, is not applicable to the phenomena of luminescence, although the qualitative rule includes more facts.

Thus, yellow flames, colored by sodium salts, with which the characteristic experiment of the reversal of the D line is made, obey the qualitative rule, but not Kirchhoff's law, properly so-called (Paschen).

I. QUALITATIVE RULE.

This relation is sometimes incorrectly defined thus: "A body absorbs radiations of the same period as those which it emits."

This definition might lead one to suppose that a body absorbs only the radiations which it emits, which is not the case. On the contrary, a body almost invariably absorbs in addition other radiations which it does not emit at all.

Consider the objects which surround us. Almost all of them appear to us to be colored,—that is, they absorb certain rays of the visible spectrum. If suitable thicknesses of these bodies are used, characteristic absorption spectra may be observed in many of them, often containing narrow bands or even absorption lines. These same bodies have at ordinary temperatures emission spectra differing among themselves, regarding which we still know nothing in the immense majority of cases, but which are limited to the infra-red region. These spectra have no known relationship with the absorption spectra observed in the visible region and in the ultra-violet.

In certain cases the absorption spectra correspond to the emission spectra which are observed, for example, when the temperature is raised; we shall soon note many examples. Shall we therefore say: A body absorbs all radiations which it is *capable of emitting*, understanding by these words the radiations which the body would emit if it were brought to incandescence, or more generally, if it became luminous? This generalization would imply an hypothesis which the facts *at present known* do not sufficiently justify.

In the statement of the rule I shall therefore specify that it applies only to radiations actually emitted, and shall formulate it (provisionally) in the following manner:

Of the radiations absorbed by a body the most important are all those

which correspond in period with the radiations emitted by the body at the same temperature and under the same conditions.

Let us see if the rule thus restricted is verified by experiment; in other words, if an appreciable absorption has always been detected for the radiations comprised in the emission spectrum observed under the same conditions.

II. VERIFICATION OF THE QUALITATIVE RULE.

1. *Line Spectra. Reversal of Lines.*

So far as line spectra are concerned, the experiments made by M. Gouy in the course of his investigations on colored flames conform directly to the proposed question. M. Gouy has in fact sought to determine whether these flames are transparent to the rays which they emit. His very simple method consists in doubling the thickness of the flame and measuring the brightness of the line by the aid of his spectral photometer. If the brightness is not doubled it is because of the fact that the emission is accompanied by appreciable absorption.

This is exactly what M. Gouy has found for all the lines which he has studied. As soon as the line has attained a certain brightness absorption manifests itself; it is, moreover, limited to the radiations comprised within the line.

In these experiments M. Gouy has examined and indicated a means of avoiding a source of error, the importance of which will be seen later, which is due to a lack of homogeneity of the flames employed. We shall shortly see that these flames are surrounded by a cooler absorbing envelope, which may appreciably modify the intensity of a ray which passes through it. For this reason these experiments may be considered, from the standpoint of the question with which we are concerned, as quite conclusive; the very parts of the flame which emit the light are those whose absorption has been studied.

May we conclude from this that there is a general rule applicable to all kinds of light? The great collection of facts connected with the reversal of lines immediately comes to mind.

The experiments of Foucault, of Kirchhoff, of Fizeau, of Cornu, etc., the numberless dark lines observed in the spectra of the Sun and the stars, seem to furnish a reply to the question thus proposed. But let us examine the matter a little more closely. It is necessary, as is well known, to distinguish two kinds of line reversals.

1. *Reversal, properly so-called*, may be observed by placing a flame giving a bright line in the path of a beam of white light giving a continuous spectrum. Under these conditions the line may appear reversed, *i. e.*, dark on a bright ground; and, if the experiment is well conducted, it may even appear as a *black line*, as though the light had actually been extinguished. It is of course understood that this is only an illusion due to an effect of contrast. The line is simply less bright than the neighboring regions. It appears black just as Sun-spots do, though their brightness is nevertheless about one tenth that of the surrounding regions, and thousands of times greater than that of the surface of the full Moon.

It will be seen without difficulty that if the qualitative rule stated above is exact, one should be able, by using a sufficiently thick flame, a sufficiently intense continuous spectrum (and a spectroscope of sufficient resolving power), to reverse *all* the lines in this way. When the thickness is increased, the beam is more and more weakened by its passage through the flame, according to the well-known exponential law. As for the brightness of the line produced by the flame itself, this *approaches a finite limit* when the thickness increases.[1] The line may thus always be rendered darker than the neighboring parts of the continuous spectrum, and will appear reversed if the dispersion is sufficient.

It is, in fact, possible, as is well known, to reverse the yellow

[1] Let e, a, be the emissive and absorptive powers, corresponding to unit thickness and to one of the radiations comprised within the line. If the total emissive power of a layer of thickness z is calculated, it is found that this emissive power approaches the limit $\frac{e}{a}$ when z increases. Thus the qualitative rule comes to this: The ratio $\frac{e}{a}$ is finite.

line of sodium; if certain precautions are taken[1] the result is very definite and striking. The same experiment was made by Kirchhoff himself, with certain lines of other alkaline or alkaline-earth metals.[2] In every case both the original bright line and the same line reversed are well seen. Finally, this is also the case with *some* of the Fraunhofer solar lines: those which correspond to the bright lines emitted by the solar atmosphere and observed by Young at the total eclipse of 1870; more particularly those which are now employed in daily observations of the prominences.

2. *Spontaneous reversal* consists, as is well known, in the appearance of a dark line on the widened bright line observed directly. First noticed by Fizeau in the sodium flame burning in air, this spontaneous reversal has acquired considerable importance since the investigations of M. Cornu,[3] who has succeeded in producing it in a great number of metallic lines in the arc or in the spark.

In the case of sodium this partial reversal is certainly produced by the cooler external envelope surrounding the flame. M. Gouy[4] has shown, in fact, that by doing away with this cooler layer the reversal is stopped. Although this cooler layer usually does not emit perceptible light, it nevertheless gives very distinct absorption lines. There is every reason to believe that the same thing is true for all spontaneously reversible lines: the vapors producing these lines are thus bodies which absorb at low temperatures the radiations which they emit under different conditions. These lines, therefore, are of great interest, since they render evident the existence of *characteristic periods*, the theoretical importance of which is considerable. But they do not furnish as convincing a reply to the very limited question which we are studying at the present time, and in which we are endeavor-

[1] In connection with this point reference may by made to an article published in *L'Eclairage Électrique*, **14**, 405 and 540, 1898.

[2] KIRCHHOFF, *Ann. Chim. et Phys.* (3), **68**, 5.

[3] CORNU, *C. R.*, **83**, 332, 1871, and **100**, 1181, 1885.

[4] GOUY, *loc. cit.*, p. 50.

ing to compare emission and absorption *under the same conditions*. It is no longer permissible here, as in the experiment of reversal properly so-called, to observe successively a bright line and the same line reversed.

Let us nevertheless assume that the vapors producing these spontaneous reversals continue to absorb when they are themselves emitting light; we shall then have a great number of lines for which the rule is valid. But if the list of substances to which these lines apply is examined, it is found that these bodies are all *metals* (or hydrogen), while up to the present time, so far as I am aware, no one has observed the reversal of a single line of a *metalloid*. This point is an important one. It is not safe to conclude, from the absence of dark lines in the spectrum of the Sun or a star, that the corresponding element is absent. It is first necessary to know whether the reversal of these lines is possible.

2. *Band Spectra.*

Does a body giving a spectrum of bands absorb the radiations comprised within these bands?

When such lines as those of sodium broaden and become diffuse, so as to resemble true bands, the absorption lines are also seen to broaden and become diffuse, in the same manner. Is it possible in the same way to reverse band spectra properly so-called?

A case in which this reversal is very distinct is that of the bands observed in the infra-red, and due to *water vapor* and *carbon dioxide*. Several investigators, Messrs. Paschen, and later Rubens and Aschkinass, have studied the absorption and the emission of these two bodies, carrying the examination down to radiations of very great wave-length. They have found that the numerous emission bands of water vapor, and the three bands of carbon dioxide observed by heating the gases directly or by studying the emission of the Bunsen flame, correspond well with very distinct absorption bands. In some cases these are so marked that the minute quantities of the gases which are always

present in the atmosphere of the laboratory, are sufficient to distinctly modify certain features of all observed spectra.

In courses dealing with the subject, two other cases are generally cited: that of erbium oxide and that of iodine vapor. Incandescent erbium oxide gives a band spectrum (Bahr and Bunsen); these bands coincide with the absorption bands which solid and dissolved erbium salts exhibit *at ordinary temperatures*. In the same way the absorption spectrum of iodine vapor is, in some measure, a negative copy of one of the emission spectra of iodine, a fluted spectrum obtained with a Geissler tube.

In the two cases the emission and absorption are not studied at the same temperature and under the same conditions. Recent investigations make possible the following comparison for iodine vapor. Iodine vapor heated to redness in a tube becomes luminous; it gives a spectrum which seems at first sight to be continuous (it is thus described by Salet and Evershed), but which, when studied more closely, appears to be formed of diffuse bands.[1]

By the use of vapors of low density M. Konen,[2] who has recently published a monograph on the spectra of iodine, has, in fact, observed in this spectrum flutings complementary to those of the absorption spectra observed at the same temperature.

Iodine vapor is therefore an example of the bodies for which the rule seems to be verified. Nevertheless, it is not yet permissible to affirm its generality. Contrary to all that precedes, certain colored flames giving emission bands seem to be wholly transparent to the radiations which they emit. This was in fact pointed out by M. Gouy for several bands of calcium, strontium,

[1] The rule connecting absorption and emission shows that if the quantity of vapor increases the emission bands (and lines) should broaden more and more, and that the spectrum should tend to appear like a continuous spectrum. The result of Salet and Evershed is explained in this way.

It should not be forgotten, however, that the particular appearance of absorption spectra depends upon the source producing the continuous spectrum which is employed in studying them; and that a very intense source causes bands to appear narrow and renders them sharper.

[2] KONEN, *Wied. Ann.*, 65, 256, 1898.

barium, copper, and carbon. Within the limits of precision of the photometric method, the brightness of these bands doubles with the thickness of the flame.

3. *Case of Fluorescent Bodies.*

On the other hand, certain fluorescent bodies obey the rule, which states that a body absorbs the same radiations that it emits, *under the conditions in which it exists.* Readers of the *Revue Générale des Sciences* have not forgotten the result of Mr. Burke's investigations,[1] which M. Guillaume has discussed with special reference to its interest from the standpoint of Kirchhoff's law. The radiations emitted by fluorescent *uranium glass* are absorbed much more strongly by this substance when it is fluorescent than when it is not.

The *qualitative rule* thus applies to uranium glass. Does it apply to all fluorescent bodies, or more generally to all luminescent bodies? Further experiments are necessary to determine this, and for many reasons Mr. Burke's very important investigations deserve to be continued and extended.

Summing up all the facts which have just been stated, we see that the rule connecting absorption and emission for the same body applies in a great number of cases, particularly to luminescent bodies, which, as will be seen later, are not governed by Kirchhoff's law properly so-called. But it is not certain that even this qualitative rule will apply in all instances; further investigations are necessary.

III. THE EFFECT OF POLARIZATION.

The statement of the rule hitherto adopted is not yet entirely correct; as we shall see, it is necessary to define it more exactly in the following manner:

If a body emits in a given direction a beam propagating certain vibrations, defined by their period and their state of polarization, it is capable of absorbing a beam propagating the same vibrations in the opposite direction.

[1] BURKE, *Proc. Roy. Soc.*, June 17, 1897; Guillaume, *Revue Gén. des Sci.*, 8, 932, 1897.

Observation of the Zeeman effect with reversed lines[1] clearly shows the necessity of limiting the statement in this way.

Suppose, for example, that the flame of a burner colored by a sodium salt is placed in a magnetic field; the lines are changed in appearance. Take, for example, the line D_2, and suppose that it is observed in the direction of the lines of force of the field. As soon as the current is established in the electro-magnet the original line disappears, and two new lines, A, B, replace it, one to the right, the other to the left of the original line. These lines are circularly polarized, in opposite directions: the more refrangible one, A, according to the rule of MM. Cornu and Koenig, is formed by vibrations corresponding in direction with the magnetizing current.

Let us now suppose that we cause a beam giving a very intense continuous spectrum to pass through the electro-magnet and the flame which it contains. When the field is cut off the line D_2 appears like a very prominent dark line. As soon as the field is established it is seen to widen, become grayish, and apparently to disappear. If a circular polarizer is placed in the path of the rays (in front of or behind the electro-magnet), or even if the observation is made through a circular analyzer, a single dark line is very clearly seen to reappear, which now occupies the place of one of the lines of the doublet observed before. If the direction of the circular polarizer is changed, the dark line is seen to take the place of the other bright line. If the field of observation is divided into two parts polarized in opposite directions, there is seen in each of these parts the line corresponding to its particular state of polarization, and only this line.

The flame in the magnetic field thus absorbs the light of the two lines A, B, but it absorbs in A only the circular vibrations having the direction of the current; in B only the vibrations of opposite direction. It is thus evident why the reversed doublet cannot be clearly seen without polarization apparatus, since the flame then affects only half of the incident light.

[1] KOENIG, *Wied. Ann.*, **62**, 240, 1897. See the article "Radiations dans un champ magnétique," *L'Eclairage Électrique*, **14**, 540, 1898.

A similar statement may be made regarding observations perpendicular to the lines of force. In this direction the flame emits three lines corresponding to D_2; it absorbs all three of them; but these lines are polarized; for each of them it absorbs only the vibrations corresponding to the state of polarization of the emitted light. In this instance the light is plane polarized, and a Nicol suffices for distinct observation of the new reversed lines. The necessity of introducing the above correction into the statement of the relation between absorption and emission is thus evident.

M. Koenig, who was the first to make these observations, has remarked on their interest from the point of view of Kirchhoff's law: "This is the first known instance," says he, "of bodies possessing a certain *anisotropie*, which is manifested both in their emission and absorption, in harmony with Kirchhoff's principle."

But Kirchhoff himself had, in fact, already made a very interesting observation bearing on the subject. As is well known, a tourmaline plate cut parallel to the axis partially polarizes a beam which passes through it. It retains this property, though in less degree, when it is heated to faint redness in the flame of a Bunsen burner. Now the light which it emits under these conditions is also partially polarized, the vibrations emitted with the greatest intensity being those which are most absorbed (Kirchhoff, *loc. cit.*, p. 186).

We shall see elsewhere in connection with Kirchhoff's law, properly so called, that other experiments also show the necessity of taking account of the direction of the vibrations in the definition of the absorptive and emissive powers.

IV. RESEMBLANCE TO RESONANCE PHENOMENA.

I cannot consider here how the qualitative rule connecting emission and absorption is to be accounted for on theoretical grounds. A reference to this subject, nevertheless, seems to be necessary.

In order to explain the existence of such a relation (partic-

ularly in the case of line spectra) it is customary to refer to the resonance phenomena studied in acoustics. The material particles are compared to vibrating systems having a definite period, and it is assumed that they are set in vibration in the manner of a resonator, by an incident wave having this period. It is recalled that a stretched cord vibrates when a cord giving the same note is bowed in the neighborhood.

There is undoubtedly a considerable element of truth in this comparison; it is evident that by imagining either entire molecules or *parts of these molecules* as capable of vibrating with characteristic periods, it is possible to explain *emission* and the qualities which characterize it.

But is *absorption* as easily understood? A resonator set in action by a sound-wave in general restores the energy which it receives in the form of vibratory energy of the same period. Is it necessary to recall the fact that in the experiments of Helmholtz on pitch the fundamental note of the resonator was heard with exceptional intensity? A ray of light, on the contrary, seems to disappear when it is absorbed, and certainly is not simply diffused in all directions by the absorbing medium.

What then becomes of the energy borne by the ray? I cannot examine here the various transformations it may undergo, which may, in fact, be predicted from theory. I desire only from the teacher's standpoint to call attention to a fact which must not be overlooked: the *resemblance* which is here established between the phenomena of optics and those of acoustics is not sufficient; it constitutes an *explanation* only when supplemented by indispensable additions.

V. KIRCHHOFF'S LAW, PROPERLY SO-CALLED.

We have seen that the ratio $\frac{e}{a}$ of the emissive and absorptive powers of a body, for vibrations defined by their period and their state of polarization, always has a finite value (which may be zero).

Kirchhoff's law, properly so-called, further states that this

ratio has the same value (which I shall designate by e_0) for *all bodies* at the same temperature. This quantity e_0 is thus a function of the temperature and the wave-length, the same function for all bodies. Let us now undertake to examine this law.

Let us remark in the first place, as M. Guillaume[1] has done, that this law does not apply to the phenomena of fluorescence. Uranium glass, for example, when exposed to very refrangible rays, emits visible radiations without sensible change of temperature; consider one of these radiations: the ratio $\frac{e}{a}$ is not zero. At the same temperature there are bodies which absorb this same radiation without emitting it; for one of these bodies the ratio $\frac{e}{a}$ is zero, equality is impossible. Kirchhoff's law, therefore, does not apply to fluorescent bodies, as compared with ordinary colored media.

It has been seen, however, that fluorescent bodies obey the qualitative rule connecting absorption and emission (Mr. Burke's experiments on uranium glass). The ratio $\frac{e}{a}$ is finite, but it is not equal to e_0; it no longer depends on the temperature and the wave-length alone.

Henceforth I shall consider that this remark can be generalized. Kirchhoff's law does not seem to apply to the phenomena of *luminescence*, *i. e.*, to cases where the emission of a given body does not depend upon the temperature alone. This is the case, for example, for colored flames, the brightness of which depends upon chemical reactions which take place within them. For these flames also the ratio $\frac{e}{a}$ is finite (reversal of lines), but it is not equal to e_0.

German physicists speak freely of an imaginary temperature called the temperature of luminescence (Wiedemann), which is attributed to luminescent bodies. According to this hypothesis luminescent bodies should also obey Kirchhoff's law, as well

[1] GUILLAUME, "L'absorption de la lumière dans les corps fluorescents," in the *Rev. Gén. des Sci.*, December 15, 1897.

as the axiom of Clausius, etc. This temperature would be that of a "black" body whose emission e_0 would be exactly $\frac{e}{a}$. But would the temperature thus calculated be *the same for all the radiations emitted?* Of this we are quite ignorant. Hitherto no one has endeavored to determine how the ratio $\frac{e}{a}$ varies with the wave-length in the case of the given luminescent body. There are reasons to believe that this ratio has nearly equal values for similar radiations, and that it is a continuous function of the wave-length; but this is all that can be said *at present.*

I shall therefore not speak of the temperature of luminescence, and, as Kirchhoff himself has done, I shall limit the application of the law to cases where the emission (of visible or invisible rays) is determined by the temperature alone; in a word, to phenomena of *incandescence.* In spite of this restriction it will be seen that the law is still of considerable importance.

In order to avoid serious error I shall recall at the outset, in accordance with the ideas of Kirchhoff, the precise definitions of emissive and absorptive power. I shall suppose at first that no account is taken of polarization phenomena; later I will return to this point.

1. *Emissive Power e.*

Consider a body C (Fig. 1) isolated in the midst of a great enclosure which neither sends to it nor returns to it radiations of any kind (a "black" enclosure at an extremely low temperature). Imagine a cylinder X meeting the body C, cutting from it an element of the surface σ. (The right section of the cylinder will have determinate but very small dimensions, which are much greater than the wave-length.)

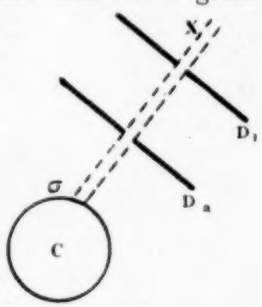


FIG. 1.

The emissive power of the body C for the radiation λ is the total intensity of the rays of wave-length

λ, which, starting from the body, are contained within the cylinder.

The number measuring this emissive power depends upon the unit of intensity adopted,[1] and upon the selection of the narrow radiation interval comprising the radiation λ, employed in the measurements. The emissive power also depends upon the cylinder X, the nature and form of the body, the condition of the surfaces—in a word, upon the entire body C, and not upon σ alone. It also depends upon the supposedly transparent medium which envelops the cylinder (S. de Smolan).

2. *Absorptive Power a.*

Suppose that *unpolarized* rays of the same wave-length λ move in a direction exactly opposite to that of the above rays, and strike the body C. Let us measure the intensity of the incident rays comprised within the cylinder, and then the intensity of what remains of these rays after they have encountered the body, *i. e.*, the intensity of the rays transmitted, reflected, or diffused by the body.[2] The difference represents the intensity of the absorbed rays; the ratio between the intensity of the absorbed rays and that of the incident rays is called the absorptive power a.

The number measuring this absorptive power for the radiation λ does not depend upon the unit of intensity chosen; it evidently cannot exceed unity. It may be very small, even when the body is quite opaque, since it depends upon the reflecting power. Like the emissive power, it depends upon X, upon the whole body C, etc.

How can we measure practically the two preceding quantities? In order to limit the beams we may imagine two diaphragms D_1, D_2, at a considerable distance apart, which will allow the passage of only the approximately parallel rays con-

[1] It is of advantage to express it in units of energy. See Guillaume, *Rev. Gén. des Sci.*, 3, 12 and 93, 1892.

[2] It is of course understood that the rays emitted by the body itself are not here included.

tained within X. We shall neglect the diffraction at the edges of the openings, and suppose that these screens neither send out nor return radiations of any kind. The beam will be allowed to fall upon a measuring instrument, which must satisfy the same conditions; unless this is the case corrections will be necessary.

It is immediately evident that the measurement of the absorptive power is more difficult, since it is necessary to measure the intensity of the transmitted, reflected, and diffused rays. In order to simplify the matter, I shall suppose that the body has the form of a plate, with plane parallel faces, and that these faces are highly polished and reflect perfectly. The reflected and transmitted rays are thus confined to limited beams; it then becomes easily possible to measure the intensity of the incident beam, and next that of the reflected and transmitted beams. If the plate is opaque but highly reflecting, the absorptive power is equal to $1-r$, r being the reflecting power. If we may neglect reflection at the faces of the plate, the absorptive power follows immediately from the measurement of the transmitted beam.

3. *Perfectly Absorptive Body.*

Finally, if the plate has a negligible reflecting power, and is sufficiently thick to permit no light to pass, the absorptive power is equal to unity. I shall call such a body perfectly absorptive *for the radiation considered.*

4. *Perfectly Black Body.*

A "perfectly black" body would be a body whose absorptive power is equal to unity for all radiations.

Do bodies exist which satisfy this condition? The question is a very important one, from several points of view. It particularly deserves to be examined carefully in connection with the study of the distribution of energy in spectra. I cannot consider it at the present time, but limit myself to the statement that arrangements have been devised which, in spite of the specific properties of the substance employed, closely represent perfectly black bodies.

For the study with which we are now occupied it is not *necessary* to possess perfectly black bodies, "*i. e.*, "black" for the whole extent of the spectrum; it is sufficient if there are bodies which absorb perfectly all the radiations which are being studied.

VI. CONSEQUENCES OF KIRCHHOFF'S LAW.

Kirchhoff's law teaches us that the ratio $\frac{e}{a}$ of the two quantities just defined remains the same when the body C is replaced by any other body at the same temperature. The constant value of this ratio is equal to the emissive power e_0 of a perfectly absorptive body.

The following consequences may be at once deduced from this law:

1. Suppose that the body C is simply rotated, without being replaced by another body; the ratio $\frac{e}{a}$ does not change. Thus we are able to deduce at once, in the special case of opaque bodies, laws defining the change of emission with the incidence. Let us suppose that the body, which is sufficiently thick to permit none of the radiations considered to pass, is bounded by a plane surface, and that without disturbing the diaphragms which limit the beam, this surface is more or less inclined (Fig. 2); the ratio $\frac{e}{a}$ must remain constant. The area cut out of the surface by the cylinder X, which limits the beams, varies (in the inverse ratio of the cosine of the angle of incidence) so that the emissive power applies to changing surfaces.

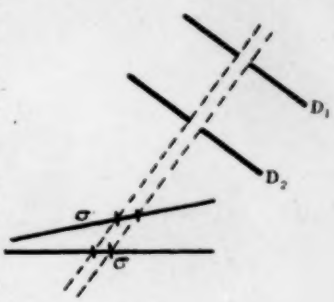


FIG. 2.

If the body is perfectly absorptive, $a = 1$, e remains constant, and the emissive power *per unit of surface* varies as the cosine of the angle of incidence; in other words, *Lambert's law is applicable*

to perfectly absorptive bodies. (Leslie found this true for lamp-black.)

If the body is opaque, but diffuses the light or reflects it regularly, the absorptive power is $1-r$, r being the reflecting or diffusing power. The emissive powers of the two surfaces σ, σ', will be connected by the relation:

$$\frac{e}{1-r}=\frac{e'}{1-r'}. \qquad (1)$$

Lambert's law will thus be applicable only if $r=r'$, *i. e.*, if the body has a *perfectly mat surface*, if it diffuses equally, whatever be the incidence. (Möller's experiments have shown that this is really the case.)

Finally, if the body is polished and highly reflecting, the reflecting power changing with the incidence, formula (1) then indicates how the emission varies with the incidence. Uljanin,[1] to whom we owe a recent investigation upon this subject, has shown that this formula well represents the experimental results, for example, those obtained with highly polished platinum.[2]

It is seen that *Kirchhoff's law, confined to the case of a single body, permits the deduction of the laws of oblique emission.* All the peculiarities of the emission of opaque bodies are known in advance when the surface reflection has been studied.

Let us pass to other consequences of Kirchhoff's law, this time comparing different bodies.

2. Let us compare two bodies which are perfectly absorptive for the same wave-length λ at the same temperature. They have the same emissive power, e_0, *for this wave-length.* Two perfectly "black" bodies give emission spectra which are absolutely identical at all temperatures.

We are acquiring a clear conception of the emissive power e_0 of a black body as a function of the temperature and of the wave-

[1] ULJANIN, *Wied. Ann.*, **62**, 528, 1897.

[2] The experiments which have just been cited were not made with monochromatic light, but it will be seen later that Kirchhoff's law may be extended to include the case of complex beams. Formula (1) is rigorously exact for every radiation and for every class of vibration considered independently.

length. The laws which have been derived (Lummer and Pringsheim, Paschen, Wien, etc.), are simple, as Kirchhoff foresaw. I shall not consider them here.

This value, e_0, which represents the emissive power of a perfectly absorptive body for wave-length λ, is the maximum value which can be attained by the emissive power e of any body whatsoever at the same temperature. We have, in fact, $e = ae_0$, and a is less than unity if the body is not perfectly absorptive.

The following statement evidently applies to any group of radiations: the total emissive power cannot exceed the emissive power of a body which completely absorbs all radiations. We shall shortly encounter an application of this statement.

3. Let us consider the entire group of radiations emitted by a "black" body at a given temperature. No body whatsoever, at the same temperature, can absorb any one of these radiations without simultaneously emitting it, and the intensity of the emission increases with the amount of absorption.

It is evident that Kirchhoff's law applies much more completely and precisely to the relationship between absorption and emission than the qualitative rule previously studied. From this rule no conclusion regarding the existence of a corresponding emission could be drawn from the presence of an appreciable absorption. Kirchhoff's law makes it possible to do this, provided that the radiation considered be emitted by a black body at the same temperature.

From this it immediately follows that the peculiarities of the absorption spectra[1] of incandescent bodies *should reappear in the corresponding emission spectra.* A colored body which absorbs certain visible rays in preference to certain others, and which preserves this property at high temperatures, should itself emit colored light. We can hardly cite investigations on this subject, other than those of Nichols and Snow, on certain oxides which

[1] By the term absorption spectra I mean the curve representing the variations of Kirchhoff's absorptive power a, with the wave-length. It is only when the reflecting power can be neglected that this curve can be deduced from observation of the transmitted beam alone.

emit colored light when incandescent. But other examples might easily be found, I believe, by studying the light emitted from *colored* glasses or fused substances heated to incandescence. I have assured myself by certain experiments that these bodies emit radiations differing in intensity and composition; Kirchhoff's law permits the properties of these emission spectra to be deduced from the properties of the absorption spectra.

What I have just said of light evidently applies to infra-red radiations, and the examples which might be cited are much more numerous. I shall refer to some of these in connection with the measurement of radiant heat, and now confine myself to the following remark: Among the radiations emitted by an incandescent "black" body, infra-red radiations occur. No incandescent body whatsoever can absorb these radiations without emitting them; consequently, in searching for an economical illuminant *if it is desired to obtain light without heat, by simple incandescence, sources which absorb but slightly in the infra-red should be employed.*

In this connection it is very remarkable that *magnesia*, which is used in the oxyhydrogen flame for the production of the Drummond light, is at ordinary temperatures exceedingly transparent for certain heat rays (K. Ångström). The same thing should be true of the mantle of the Auer burner, if, as M. Le Chatelier believes, the light from this source is due to simple incandescence. It might be investigated directly whether the incandescent mantle absorbs but slightly in the infra-red,[1] and whether it obeys Kirchhoff's law.

VII. HAS KIRCHHOFF'S LAW BEEN VERIFIED EXPERIMENTALLY?

The consequences which we have just deduced from Kirchhoff's law seem to be in accord with the observed facts; but has this law, in the form in which it has been announced, *i. e.*, for a *single wave-length*, been directly verified in a rigorous manner?

[1] More exactly, that part of the infra-red which is the most intense in the spectrum of a black body. The mantle of the Auer burner (of course with its glass chimney removed) is particularly rich in the recently studied heat radiations of 50–60μ.

Let us first examine the experiments made on the discontinuous spectra of gases. From our present point of view, gases offer the advantage of having a negligible reflecting power. Moreover, it seems natural to examine them first, since they give those narrow lines of which the physicist often speaks as though they were formed of a *single* radiation.

It is known that these lines have in reality a variable width and a complex constitution; this point has been made clear by the experiments of M. Gouy, and those of Professor Michelson. It is impossible to regard them as formed of a single radiation, and it could not even be assumed that if a portion of these lines were made to pass through a slit used in place of the eyepiece of a spectroscope, the intensity could be considered as uniform within this very narrow interval. We thus encounter immediately the following question: Can Kirchhoff's law be verified by means of experiments on complex radiations?

This question will be examined later. For the moment let us consider how Professor Paschen[1] has endeavored to determine whether Kirchhoff's law is applicable to flames and to incandescent gases.

He has utilized the statement already referred to, according to which the emissive power of a body obeying Kirchhoff's law can never exceed the emissive power of a body which at the same temperature completely absorbs *the radiations considered.*

1. *Arc Lines.*

In collaboration with Professor Kayser, Professor Paschen has compared by the aid of photography the brightness of an ultra-violet region of the continuous spectrum given by the positive crater, with the brightness of bands belonging to carbon and magnesium, situated in the same region, and given by the vapor of the same arc. The brightness of these bands is much greater than the brightness of the positive crater, which is nevertheless at a higher temperature, according to M. Violle's measures. If we assume (as Professor Paschen does implicitly) that

[1] PASCHEN, *Wied. Ann.*, 51, 41, 1894.

the carbon at this temperature is capable of absorbing ultra-violet rays ($\lambda = 3800$), it follows that Kirchhoff's law is wholly insufficient. Professor Paschen therefore concludes that these bands are produced by a luminescence phenomenon.

2. *Yellow Sodium Lines.*

Professor Paschen measured with the spectrobolometer the total intensity of the two D lines given by the flame of a Bunsen burner, and then, other things being equal, the corresponding intensity of a selected region in the spectrum of a black body, a region completely including the two D lines. Assuming an excessive value for the width of the sodium lines, Professor Paschen then calculated a maximum value of the intensity of the part of the spectrum of the black body corresponding to the lines. This value is not *even half* the value found with the lines given by the burner. Kirchhoff's law is, therefore, inapplicable, and Professor Paschen concludes that the brightness of these lines is due, at least in large part, to a phenomenon of luminescence.

The "black" body employed was simply a sheet of platinum, heated by a current to the temperature (1470°) found for the flame. It may be asked whether the platinum is really a perfectly absorptive body under these conditions. Professor Paschen has the right to assume from other investigations that its emission *in this region of the spectrum* and at this temperature, is sensibly as great as that of lampblack, which with certain precautions can be studied at this high temperature. The question is thus reduced to the following: "Is lampblack at this temperature a "black" body and a perfectly absorptive body?" Professor Paschen considers it possible to affirm, after all of his investigations on emission, that it is very nearly so. It should not be forgotten, moreover, that an excessive value was calculated for the emission of platinum, that the flame was not very thick, and that consequently a more precise measurement would have accentuated the departure from Kirchhoff's law.

Wiedemann, using a different method, arrived at the same

conclusion. Moreover, the direct investigations of Evershed, Pringsheim, etc., clearly show that chemical reactions are necessary to produce the very bright lines of flames, and that the sodium line is less brilliant when the vapor of the metal is directly heated, all chemical action being, as far as possible, prevented.

Flames colored by sodium serve well to exhibit the reversal experiment in lecture courses on this subject: no experiment better demonstrates the necessity of distinguishing Kirchhoff's law from the rule connecting the absorption and emission of a given body.

3. *Infra-red Band of Carbon Dioxide.*

On the other hand, the emission of heated carbon dioxide seems to be due to temperature alone. Professor Paschen has studied the very sharp band in the infra-red near $\lambda = 4.3\mu$, which is given by this gas. By heating in a tube a layer of the gas 7 cm thick, the maximum of the observed band is a little below but not far from the curve which represents the emission of lampblack under the same conditions. Further, Professor Paschen has profited by the fact that this band is very marked in the emission spectrum of the Bunsen flame, in order to test the matter at a higher temperature, arranging the experiment as in the case of the D lines. It is necessary, however, to take into account the fact that platinum in this region is far from being perfectly absorptive. This experiment is not susceptible of great precision, and Professor Paschen does not hold it to be so, *but the law seems to be verified in the case where the emission is determined by the temperature alone.*

This is the only attempt which has been made up to the present time to verify Kirchhoff's law in the form in which it has been stated; *i. e.*, for a *very narrow* region in the spectrum. *No other trial has hitherto been attempted with luminous or ultra-violet radiations.* The experiment would nevertheless be possible by the use of incandescent solids or liquids, and the remarks which follow should facilitate its performance. There are many reasons to believe that it would give an affirmative response.

VIII. KIRCHHOFF'S LAW EXTENDED TO A GROUP OF RADIATIONS.

It has frequently been stated that Kirchhoff's law has been verified by means of experiments on radiant heat. How can experiments made on the numerous radiations comprised in infra-red spectra apply to this law, which refers to a single wave-length.

Let us see in a general way whether it is possible to extend Kirchhoff's statement to a *group of radiations comprised between two definite limits*. One might be tempted to assume *a priori* that what is true for each radiation must be true for the group. Nevertheless, this is not the case, and I think it important to insist upon a condition which is necessarily imposed, and which it would be incorrect to leave out of account.

The total emissive power E for a complex beam may be at once defined as the total energy of the beams corresponding to each of the radiations. The total absorptive power A may also be defined as the ratio between the absorbed energy and the energy of an incident beam. For a single wave-length this ratio is perfectly definite, and is entirely independent of the beam employed in measuring it, the intensity of which is arbitrary. But if it is a question of a group of radiations for which the absorptive power of the body under consideration does not remain constant, the number obtained for A will depend upon the distribution of the energy in the spectrum of the chosen beam. It will change with the source employed in the measurement, since the various emission spectra are not identical, and the ratio $\frac{E}{A}$ will not even have a constant value for a single body.

The same beam should therefore be selected at the outset for the measurement of the absorptive power of all bodies. *But this is not sufficient.* If we have two bodies, C, C', for which $\frac{E}{A}=\frac{E'}{A'}$,[1] it is not sufficient to take a certain arbitrary beam for measuring A, A'. But it is easily seen that these two ratios are

[1] The two ratios are equal if the beam sent out by C' is employed in the measurement of A, and reciprocally the beam coming from C in the measurement of A'.

equal when they are measured *by the use of a beam derived from a body at the same temperature which perfectly absorbs all the radiations of the beam.*[1]

If this condition is fulfilled the ratio $\frac{E}{A}$ is the same for *all* bodies and equal to E_0, the total emissive power in this region of the spectrum, of a "black" body. Or again, the *relative emissive power* $\frac{E}{E_0}$, corresponding to a black body, is equal to the absorptive power thus defined.

(I have supposed that use can be made of a measuring instrument accurately indicating the energy received for all the radiations studied. It is immediately evident that if this condition is not fulfilled the values found must still satisfy the same relation, provided of course that the instrument remains the same in all the measurements.)

Let us apply what has just been said to the entire heat spectrum. We thus come again upon the statement of the law known since Leslie, but with an *indispensable* complement. It is in this way that the various investigations on radiant heat, concerning the equality of the relative emissive and absorptive powers, may be referred to Kirchhoff's law.

If these investigations are examined it is found that the conditions indicated above are not satisfied. Most frequently the source used in measuring the absorptive power is at a temperature much higher than that of the body under consideration; this source frequently differs widely from a black body; finally, absorption is almost invariably studied at lower temperatures than those employed in the investigation of emission.

[1] Let e_0 be the emissive power of a black body for the radiation λ; we have, taking the integrals between the chosen limits,

$$E = \int e\, d\lambda = \int a\, e_0\, d\lambda,$$

$$A = \frac{\int a\, e_0\, d\lambda}{\int e_0\, d\lambda};$$

$$\text{hence } \frac{E}{A} = \int e_0\, d\lambda = E_0.$$

The measures which De la Provostaye and Desains have published on this subject, in their valuable collection of investigations on radiant heat, are the only precise ones which can be cited. They refer more particularly to opaque reflecting bodies.[1] The preceding criticisms are here of no great weight: the reflecting powers for the infra-red of the bodies studied (gold, platinum, glass, under various angles of incidence), change but little with the temperature; the same thing is true of the absorptive powers. Moreover, the sources used in the measurement of these reflecting powers are at temperatures but slightly higher than those of the mirror, and are covered with lampblack. Thus the test could be made with a satisfactory result.

As much cannot be said for the other investigations; most frequently they have furnished an ample harvest of interesting facts, which certainly tend to support Kirchhoff's law, but which cannot be considered to verify it rigorously.

In this category may be placed those experiments which result in the formation of lists of bodies arranged in the order of their increasing emissive powers (relative), and then in the order of increasing absorptive powers, which turn out to be almost identical. It is doubtless interesting to note that polished metals, for example, which are highly reflecting, and which have a low absorptive power, also have a low emissive power; but it is unnecessary to give the lists prepared on this subject; the order of the substances, classed according to their increasing absorptive powers, varies, in fact, with the nature of the source. I shall nevertheless mention Tyndall's experiments, because his very important results are less widely known and are instructive in other ways: gases have emissive and absorptive powers in the infra-red which are nearly proportional; these powers are very different for different gases; gases formed of coarse "molecules" give far higher values than simple gases.

Ethylene, for example, absorbs much more strongly than atmospheric air; it also emits more strongly, and the emission

[1] Other less direct tests have been made for rock salt and for a diffusive body, lead borate.

of a polished plate of metal can be considerably increased by coating it with a film of this gas. In the same way nitrogen and hydrogen show a hardly perceptible absorption; combined in the form of ammoniacal gas their absorption or emission becomes considerable. I cannot do better than refer to the numerous experiments on this subject given in Tyndall's work (*Heat*); many important facts will be found there which support Kirchhoff's law, particularly the experiments showing that those gases which emit discontinuous spectra have marked absorptive powers when the source gives out radiations which exactly correspond to them.

IX. KIRCHHOFF'S LAW AND POLARIZATION BY EMISSION.

Up to the present no account has been taken of polarization phenomena. If these phenomena are taken into consideration, as was done by Kirchhoff himself, the statement of the law can be made still more precise and other experimental facts can be referred to it.

Let us suppose that in the measurement of the emissive power the entire emitted beam is not considered, but only a part of it—that which is transmitted by a given analyzer (plane or circular). In passing out from the analyzer the beam propagates certain well defined vibrations; its intensity measures the emissive power corresponding to the radiation and *to the vibrations* considered. For example, with a plane polarizer there will be obtained the emissive power e_p, corresponding to vibrations of direction p which are allowed to pass by the analyzer.

Moreover, the corresponding absorptive power a_p will be obtained by bringing to the body under investigation a beam *propagating the same vibrations* (obtained with the same apparatus, which in this case acts as a polarizer).

Kirchhoff's law is then as follows: The ratio $\frac{e_p}{a_p}$ of the emissive and absorptive powers, corresponding to radiations of definite period and to definite vibrations, is the same for all bodies at the same temperature. It does not depend upon the particu-

lar kind of vibrations selected, or, if the polarization is plane, upon the orientation of p.

When a body emits partially polarized light, which is the general case, the emissive power e_p is not the same for all the vibrations; the absorptive power consequently also depends upon the kind of vibrations considered. It is thus seen why, in the definition of the emissive and absorptive powers e, a, measured without polarizing apparatus, I have assumed that the beam used in the measurement of a was of common light: the law as stated for e and a is rigorous *in all cases*,[1] but only under this condition.

Among the experimental facts relating to the law in its rigorous form I shall again mention Kirchhoff's experiment on tourmaline, which, when heated to redness, emits most strongly those vibrations which it most freely absorbs. If this observation were repeated quantitatively the measurements would doubtless result in a verification of the law. An analogous experiment would undoubtedly succeed with liquids, which absorb in different degrees two rays circularly polarized in opposite directions; these liquids, if heated, should emit heat rays which are partially circularly polarized; those which in the infra-red absorb *right-handed rays* most freely, should most readily emit *left-handed rays*, for these two kinds of rays propagate the same vibrations in opposite directions.

The phenomena of *polarization by emission*, in the case of bodies having reflecting power, are particularly important in affording a precise verification of the above law. From this law it follows immediately:

1. That the rays emitted by a "black" body are never polarized.

2. That if there is partial polarization by reflection, there is partial polarization in a plane at right angles at emission.

3. That for rays emitted or reflected in a given direction the ratio $\frac{e}{1-r}$ is the same for vibrations parallel and perpen-

[1] It is immediately deduced from the law which has just been stated, for example, by separating a beam of ordinary light into two beams polarized in planes at right angles.

dicular to the plane of incidence, if the body is opaque and perfectly polished.

These facts are rendered evident by experiments made before Kirchhoff's investigations. The merit of having extended to radiant heat the discovery of polarization by emission, made by Arago, and of having shown the close relation connecting this phenomenon with polarization by emission, in fact belongs to De la Provostaye and Desains.[1] Concerned more particularly with the thermal equilibrium in enclosed spaces, De la Provostaye and Desains state this relation in other terms,[2] but their experimental results permit the law to be verified in the form in which it has just been stated (experiments on polished platinum for heat and light, on glass for obscure heat).

If we add that Kirchhoff's law makes possible not only the prediction of polarization by emission, but a prediction of how it should vary, for example, with the incidence,[3] it is evident how it permits a relation to be established between apparently very different phenomena and gives a means of discovering their laws. Other cases in which it could render similar services might easily be found.

X. KIRCHHOFF'S LAW AND TEMPERATURE EQUILIBRIUM.

It is a fact admitted by all that temperature equilibrium once established within an enclosed space, protected against all external radiation, would persist indefinitely. This maintenance of equilibrium may be considered as an experimental fact, or deduced from Clausius' axiom.

Now Kirchhoff's law may be derived from this fact by making a certain number of hypotheses, which must be enumerated:

1. We assume Prévost's theory of exchanges, *i. e.*, we sup-

[1] For their investigations on radiant heat see Desain's *Leçons de Physique*, 1860.

[2] "In an enclosed space in equilibrium partial polarization by emission completely compensates the effects of polarization by reflection, so that all rays in the enclosure are unpolarized."

[3] See Uljanin (*loc. cit.*), who confirmed in this way the results of M. Violle's experiments on molten silver.

pose that each part of the enclosure receives and emits radiations, even when the temperature is uniform. This is an hypothesis, for *at present* we have no means of determining the existence of such rays ; we have no method of studying a radiation without causing it to disappear.

2. We assume that this incessant radiation explains the preservation of the equilibrium, and that the other modes of propagating heat (conduction and convection) are not concerned. This is also an hypothesis ; these modes of propagation play an important part in establishing the equilibrium.

3. We assume that the emission is determined for a given body by the temperature alone, *i. e.*, that cases where radiation results from chemical action, fluorescence, or any other luminescence phenomenon, are not included.

4. Conversely, we assume that an absorbed radiation is wholly transformed into heat, *i. e.*, produces merely a rise of temperature. It must not produce chemical action, fluorescence phenomena, etc.

Such are the hypotheses, though they have not always been explicitly stated, which have been made by all investigators who have endeavored to connect the relationship between absorption and emission with temperature equilibrium. I cannot consider here researches prior to Kirchhoff, in which the equality of the emissive and absorptive power (relative) were thus demonstrated for thermal radiations in general, nor dwell upon the importance of the investigations of De la Provostaye and Desains. This proposition was not Kirchhoff's law, which applies to a *single* radiation.

Kirchhoff has himself endeavored to deduce the law in the precise form in which he announced it, from temperature equilibria and from the preceding hypotheses. He has given two successive demonstrations. In the first he imagines two plane surfaces of indefinite extent, one of them perfectly black, the other covered with a substance *which emits and absorbs only one radiation.* Provided that the temperature does not change, he concludes that, for this radiation, the ratio of the emissive and

absorptive powers of this substance is equal to the emissive power corresponding to a black body.

This demonstration, which is too frequently reproduced in the classic works of the present day, does not establish the law rigorously, nor in its most general form. The most radically defective point, in my opinion, is the assumption on *a priori* grounds of an imaginary body which would emit but one radiation, and would absorb this radiation and no other.

The other demonstration which Kirchhoff gave soon afterward[1] is more rigorous and complete. I cannot outline it here; it is long and complicated, principally I suppose, because Kirchhoff took into account polarization phenomena. It certainly might be simplified.[2] I confine myself to mentioning the ingenious device by the aid of which he showed that equilibrium must be established for each kind of radiation considered by itself; he supposes a thin, perfectly transparent plate, which gives the colors of thin plates and reflects certain radiations to the exclusion of certain others, to be suitably placed in the enclosure which is being studied. As the equilibrium must hold for all thicknesses of the plate, a simple calculation shows that it must exist for each radiation.

These demonstrations in which imaginary bodies are employed (bodies perfectly absorptive for negligible thickness, perfectly transparent or totally reflecting bodies which neither emit nor absorb at any temperature) may be considered as far removed from ordinary experience. Nevertheless, these imaginary bodies may be realized with a higher and higher degree of approximation, and this renders their use legitimate. Moreover, the suggestive value of such reasoning must not be overlooked; the example of Desains and De la Provostaye shows that new facts and the laws which control them can be discovered in this way.

But such reasoning does not constitute a "theory" of Kirch-

[1] *Ann. Chim. et Phys.*, **67**, 160, 1861.

[2] A Nicol might be imagined in the enclosure. See in the memoir of De la Provostaye (*Ann. Chim. et Phys.*, **67**, 5,) the demonstration which is given for regularly reflecting bodies.

hoff's law, for it does not connect this law with the generally accepted theories of light. Is such a theory possible at the present time?

In the facts which have just been studied, the emission and absorption have been seen to be modified both by the superficial properties of the bodies and by their molecular structure. If an attempt is made to account for the relations found, one is led either to construct the theory of reflection itself, or to study the relationship of the ether to material molecules. In order to show this I shall examine only two special cases.

Let us consider *opaque reflecting* bodies. As has been seen, Kirchhoff's law connects the emissive and reflecting powers. Now it may be connected directly with Helmholtz's theorem concerning reflection, according to which the reflecting power is independent of the direction of propagation of a ray undergoing refraction while passing from one medium to another. It is sufficient to assume, with Fourier and many others since his time, that the radiation does not take place from the surface alone, but from a layer of greater or less depth. If the surface did not interfere, the radiation would be that of a perfectly absorptive body; but it returns a part of the rays toward the interior, and this reflection weakens the beam, partially polarizes it, in a word, endows it with all its characteristic properties.

Let us now consider two bodies which are perfectly absorptive for a given radiation. In spite of the great differences which they may present, particularly as regards their chemical structure, these two bodies, at the same temperature, have the same emissive power for the radiation considered. Near their surface, the amplitude of the vibrations of the ether having the chosen period is perfectly definite, and depends only on the temperature and the period. Such is the important consequence of Kirchhoff's law.

From this it may be seen how this law, which connects together so many experimental facts, brings an important contribution to the theoretical study of the relationship between ether and matter, which is still so mysterious.

Minor Contributions and Notes.

PHOTOGRAPHS OF THE NEW STAR IN SAGITTARIUS.

The photographs of Nova Sagittarii reproduced in Plate III were taken at the Cambridge and Arequipa stations of the Harvard College Observatory. The chart plate shows the star as it appeared on April 29, 1898, when its magnitude was 8.4. The first spectrum plate, taken at Arequipa on April 19, 1898, shows the lines $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, $H\eta$, and probably $H\theta$, bright. A broad band at $\lambda 4643$ is also bright, and several narrow bright lines are shown. These lines appear to be identical with the corresponding lines found in the spectrum of Nova Aurigae. The spectrum plate taken two days later reveals certain important changes, notably the appearance of a narrow bright line at $\lambda 5005$, which probably coincides with the chief nebular line at $\lambda 5007$. Further information regarding the photographs may be found in *H. C. O. Circular* No. 42, published in the last number of this Journal, from which this description has been taken.

NOTE ON THE NEW FORM OF PHOTOGRAPHIC TELESCOPE PROPOSED BY PROFESSOR PICKERING IN H. C. O. CIRCULAR, No. 39.

As it seems desirable that the special advantages and disadvantages of the form of telescope which is proposed by Professor Pickering, in *H. C. O. Circular* No. 39, should be carefully considered, before the construction of such an instrument is undertaken, I venture to suggest that the focal length proposed in the circular is somewhat excessive. It is very doubtful whether anything would be gained by increasing the focal length beyond the point which gives full photographic resolution. The resolving power of an objective depends on its aperture only, and for visual purposes the best ratio of focal length to aperture is determined by practical considerations. In photographic optics, the size of the silver grains on a negative plays an important part.

A telescope which has the usual ratio of focal length to aperture (say 15 : 1 or 20 : 1) gives an image in which the detail is too fine for

the photographic plate; but if, with the same aperture, we make the focal length very great (say 200 : 1), we obtain an image in which these relations are reversed. Somewhere between these limits, therefore, is a value of the focal length for which image and plate are suitably related.

This question has been treated by Professor Wadsworth,[1] who took as the basis of his discussion the condition that the two maxima and intervening minimum of a double-star image just at the point of resolution should fall on separate and distinct silver grains. The average distance between the silver grains on a negative was determined by measurement. Professor Wadsworth's conclusion was that the focal length of a photographic telescope should be about thirty-five times the aperture.

In view of the uncertainty which attaches to the definition of photographic resolution, the most suitable ratio of focal length to aperture cannot be very exactly fixed, but it doubtless lies between the limits 30 : 1 and 60 : 1.

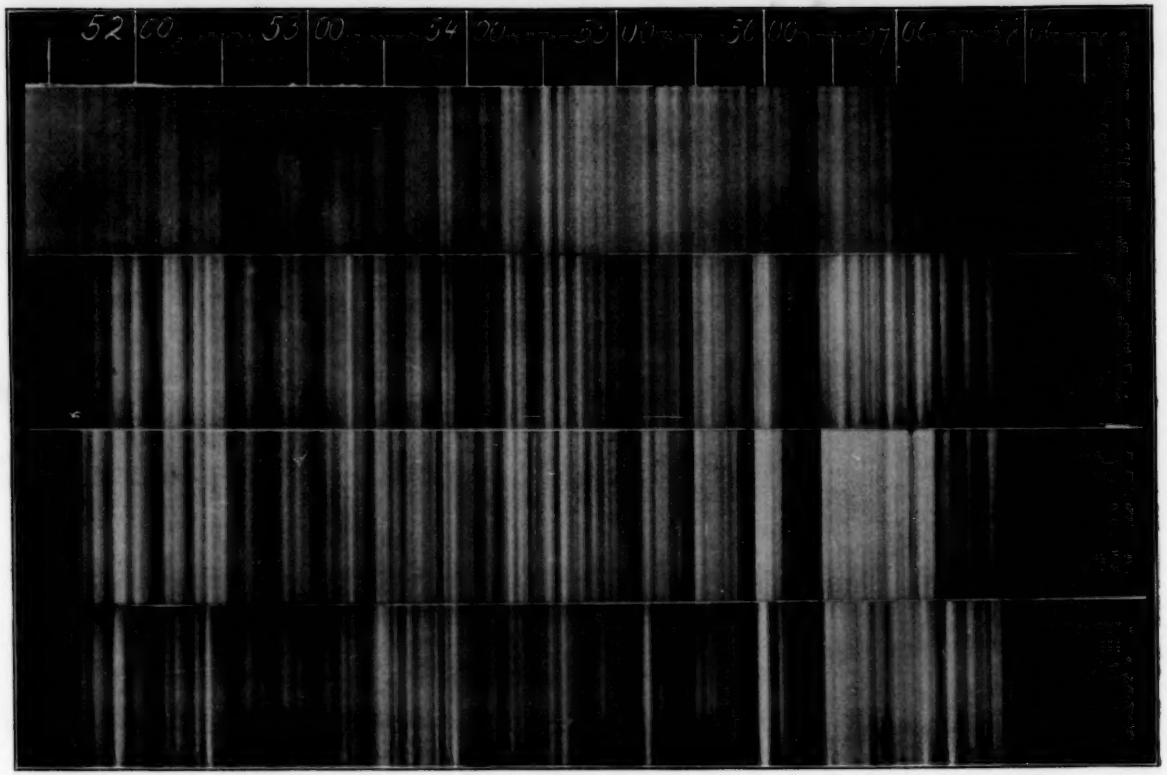
This conclusion is borne out by experience. At the Allegheny Observatory I found that a spectrograph camera, which was designed for photographing the spectra of bright objects, and which had a focal length twenty-seven times greater than the aperture, would photograph nearly, but not quite, all the detail in the solar spectrum that the eye could see. At the Lick Observatory, the best results in lunar photography with the great equatorial have been obtained with an aperture of about 12 inches ($F = 48$ A). The large concave gratings of Professor Rowland give full photographic resolution, with $F = 44$ A. The ratio for the horizontal photoheliograph of the Lick Observatory ($F = 96$ A) is probably greater than necessary (from the present point of view), particularly since fine-grained plates are used with this instrument.

I have seen photographs of the Moon and planets taken with telescopes of very great (virtual) focal length, and their appearance was just what the foregoing considerations would lead one to expect. The enlargement had been carried altogether too far.

To secure the advantages pointed out by Professor Pickering, it would be well to give the focal length the benefit of every doubt, but in my opinion it should in no case exceed 100 times the aperture. If carried beyond this limit, nothing would be gained, while the greater

[1] This JOURNAL, **3**, 188, 1896.

PLATE IV.



SPECTRA OF STARS OF SECCHI'S TYPE IV.

I. 280 Schjellerup = DM. 59°2810 (Mag. 7.8).
II. 273 Schjellerup = 19 Piscium (Mag. 5.5 ±).
III. 132 Schjellerup = U Hydrae (Mag. 5.5 ±).
IV. 152 Schjellerup (Mag. 5.5).

length of exposure required, and the increased atmospheric disturbances within the instrument, would be positive disadvantages.

JAMES E. KEELER.

THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO.

BULLETIN NO. 7.

SPECTRA OF STARS OF SECCHI'S FOURTH TYPE.

SOME of the results of a photographic study of the spectra of stars of Secchi's type IV (Vogel's III*b*), upon which Mr. Ellerman and the writer have been engaged for over a year, were presented at the Harvard Conference last August (see ASTROPHYSICAL JOURNAL, **8**, 237, 1898). Since that time, on account of the substitution of a train of three prisms and a short camera ($f = 10.8$ inches) for the single prism and longer camera ($f = 20.0$ inches) used with the spectrograph in most of the earlier work, much better photographs have been obtained with shorter exposures. Four of these spectra are reproduced herewith. The order of arrangement corresponds with that mentioned in the paper referred to above, in which it was stated that it had been found possible to group ten stars in a series. This grouping presumably represents the normal order of development. The spectra of other stars in the series might also have been reproduced, but the four selected may be taken as fairly representative of well-defined steps in the evolutionary process. The individual spectra, which will shortly be illustrated in more detail, and discussed in connection with the measures of the wave-lengths of the bright and dark lines, are of present interest in their bearing upon Professor Dunér's important paper "On the Spectra of Stars of Class III*b*" in the March number of the ASTROPHYSICAL JOURNAL.

The presence of bright lines in the spectra of these stars, announced by the writer at the Harvard Conference, is now abundantly confirmed. Some of the more conspicuous of these lines were seen by Professor Dunér with the Upsala refractor before the photographic study of the spectra was undertaken here. They have recently been observed visually by Professors Keeler and Campbell at the Lick Observatory. In any attempt to connect these spectra with those of other types the bright lines must not be overlooked.

The difficulty of establishing such a connection cannot be said to have materially lessened. No star intermediate in character between type IV (III*b*) and any other known type has yet been found. This is perhaps hardly surprising, as the spectrum of such a star would not be likely to exhibit striking characteristics which might lead to its early detection. No systematic search for such a star has been attempted here, as the Observatory does not possess instruments especially adapted to the purpose. Advantage will be taken of Professor Pickering's kind offer to photograph suspected objects with an objective prism. Should the character of the spectra thus obtained be such as to warrant more detailed study, the 40-inch refractor and stellar spectrograph of this Observatory will be employed for the work.

March 29, 1899. GEORGE E. HALE.

BULLETIN NO. 8.

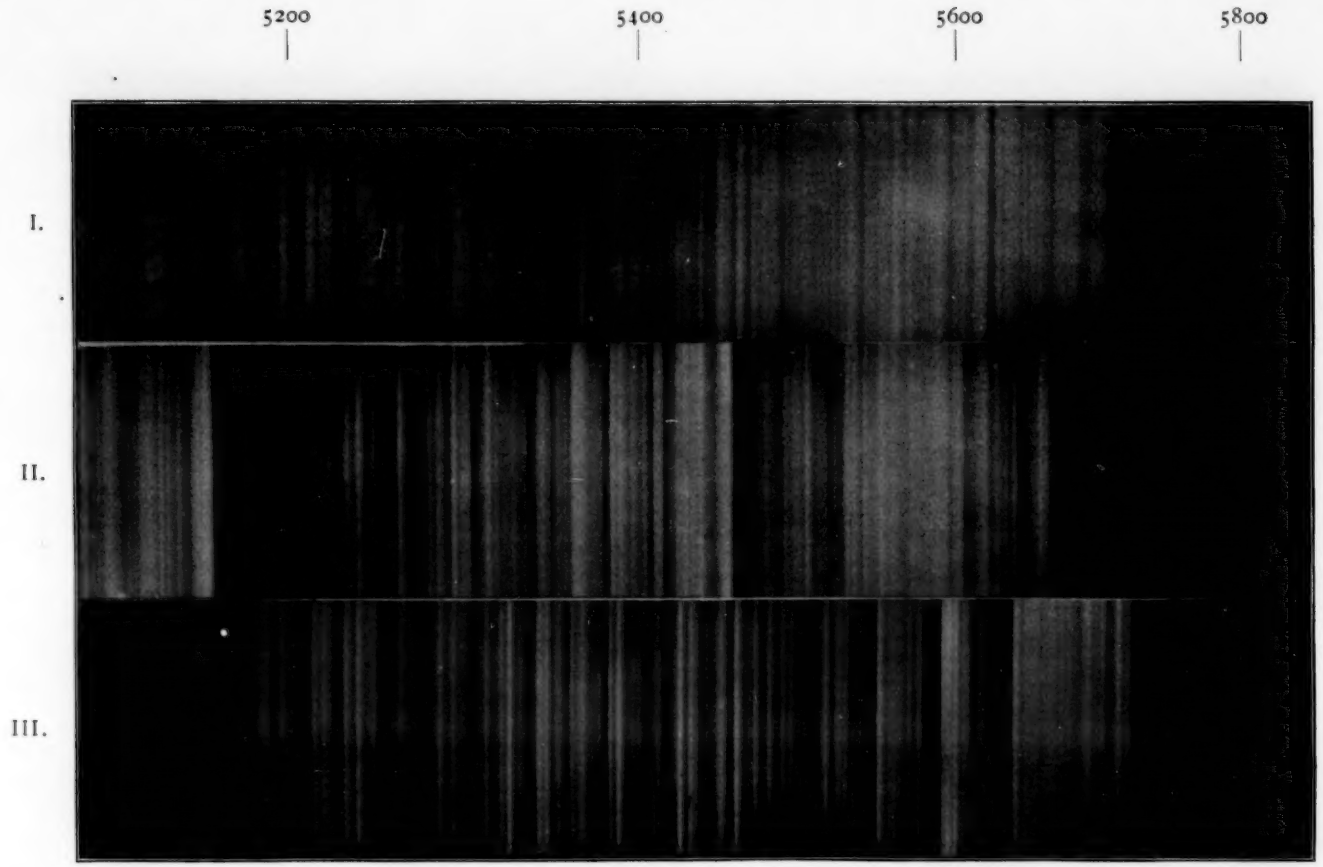
OPPORTUNITIES FOR STUDENTS AT THE YERKES OBSERVATORY.

THE attention of advanced students in astronomy and astrophysics is invited to the fact that opportunity for work at the Yerkes Observatory is offered to all who can be accommodated. Ample preparation for advanced studies in theoretical and practical astronomy and in astrophysics is afforded by the courses given at the University of Chicago in the Department of Mathematics, Astronomy and Physics. After completing the necessary preliminary work in Chicago, students who desire to devote special attention to observational astronomy or to astrophysics are admitted to the Yerkes Observatory at Lake Geneva, where they are given every possible facility. In addition to pursuing the courses of instruction enumerated in the *Annual Register* of the University of Chicago, students at the Observatory may take part in the regular work of research. As soon as they have had sufficient preliminary training they are encouraged to undertake original investigations of their own.

VOLUNTEER RESEARCH ASSISTANTS.

It may not infrequently be the case that students who have taken higher degrees in astronomy, astrophysics, or physics, or have pursued advanced studies in these subjects at the University of Chicago or some other institution, will find it to their advantage to spend some time at the Yerkes Observatory, in order to familiarize themselves with

PLATE V.



SPECTRA OF STARS OF SECCHI'S TYPES II, III, AND IV.

I. The Sun (type II).
II. μ Geminorum (type III).
III. 132 Schjellerup (type IV).

its work. To meet this need the position of Volunteer Research Assistant has been established. Those who are appointed to this position are expected to carry on such work as may be assigned to them during their connection with the Observatory. They receive no pay for their services, but are given every reasonable opportunity to become acquainted with the investigations in progress, and in some cases to conduct researches of their own. During the summer of 1898 Dr. Frank Schlesinger, Ph.D. (Columbia University), Mr. J. A. Parkhurst, S.B. (Private Observatory, Marengo, Ill.), and Mr. A. L. Colton, M.A. (University of Michigan, recently assistant at the Lick Observatory), held positions at the Yerkes Observatory as Volunteer Research Assistants. Dr. Schlesinger aided in the measurement and reduction of photographs of stellar spectra, and carried out an independent investigation of the spark spectrum of iron taken in air. Mr. Parkhurst made systematic observations of variable stars with the 12-inch refractor, and determined the distribution of stars of Secchi's fourth type with reference to the Milky Way. Mr. Colton gave important assistance to Professor Nichols in his observations of the heat radiation of Arcturus and Vega.

Further information regarding opportunities for students and the appointment of Volunteer Research Assistants may be obtained on application to the Director of the Yerkes Observatory, Williams Bay, Wisconsin.

GEORGE E. HALE.

April 3, 1899.

BULLETIN NO. 9.

COMPARISON OF STELLAR SPECTRA OF THE THIRD AND FOURTH TYPES.

IN *Bulletin* No. 7 it was stated that but little progress had been made toward establishing a relationship between stars of the fourth type and those of other classes. Some excellent enlargements from our most recent negatives, which have just been made by Mr. Ellerman, bring out in a very striking way a resemblance between stellar spectra of the third and fourth types. This resemblance was alluded to in a paper read by the writer at the Harvard Conference (see an abstract in the ASTROPHYSICAL JOURNAL, **8**, 237, 1898), but the earlier photographs, made with the dispersion of a single prism, were on too small a scale to permit any conclusions to be safely drawn. The

II. μ Geminorum (type III).
III. 132 Schjellerup (type IV).

photographs reproduced in the accompanying plate, which were made with a three-prism spectrograph attached to the 40-inch telescope, are fairly well adapted for the purposes of this comparison. It will be seen that in the region extending from b_4 to about $\lambda 5300$, the spectra of μ Geminorum (type III) and 132 *Schjellerup* (type IV) are almost identical, while there are many common lines in the neighboring less refrangible part (the slight difference in scale of the two photographs is not sufficient to render the comparison difficult). Further toward the red the spectra become very unlike, though even here there are certain important points of resemblance which must be carefully investigated. The agreement of the spectra of the two stars is quite as striking in a limited region lying between $H\beta$ and $H\gamma$. In fact, it may be said that within certain limits in both the green and blue regions the spectrum of μ Geminorum more closely resembles that of 132 *Schjellerup* than do the spectra of certain stars of the *fourth* type. It is interesting also to notice that between b_4 and $\lambda 5300$ the spectrum of μ Geminorum agrees more perfectly with the spectrum of 132 *Schjellerup* than it does with the solar spectrum (type II).

These photographs serve to confirm the common belief in the essential similarity of the two types of red stars, and may perhaps afford material for a study of their development.

GEORGE E. HALE.

April 12, 1899.

A NEW SATELLITE OF SATURN.[1]

NEARLY all of the astronomical discoveries made by the aid of photography have related to the fixed stars. In the study of the members of the solar system, the results obtained by the eye are generally better than those derived from a photograph. For many years it has been supposed that photography might be used for the discovery of new satellites, and in April 1888, a careful study of the vicinity of the outer planets was made by Professor William H. Pickering. Photographs were taken with the 13-inch Boyden telescope, with exposures of about one hour, and images were obtained of all the satellites of Saturn then known, except Mimas, whose light is obscured by that of its primary. It was then shown that Saturn probably had no satellite, as yet undiscovered, revolving in an orbit outside of that of Enceladus,

[1] *Harvard College Observatory Circular* No. 43.

unless it was more than a magnitude fainter than Hyperion. (Forty-third Report, p. 8.)

In planning the Bruce photographic telescope, a search for distant and faint satellites was regarded as an important part of its work, and accordingly, plates for this purpose were taken at Arequipa, by Dr. Stewart. A careful examination of these plates has been made by Professor William H. Pickering, and by superposing two of them, A 3228 and A 3233, taken August 16 and 18, 1898, with exposures of 120^m, a faint object was found which appeared in different positions on the two plates. The same object is shown on two other plates, A 3227 and A 3230, taken August 16 and 17, 1898, with exposures of 60^m and 122^m, respectively. The position is nearly the same on the two plates taken August 16, but on August 17 it followed this position $33''$, and was south $19''$, while on August 18 it followed $72''$, south $43''$. Its motion was direct, and less than that of Saturn, though nearly in the same direction. It cannot, therefore, be an asteroid, but must be either a satellite of Saturn or a more distant outside planet. The proximity of Saturn renders the first supposition much more probable. On August 17, the position angle from Saturn was 106°, and the distance $1480''$. Assuming that it was at elongation, and that its orbit is circular, its period would be 400 days, or five times that of Iapetus. It was at first identified with a very faint object found on plates taken in 1897, and the period of seventeen months was derived from them. This supposition has not been confirmed.

Measurements of the positions of the images give additional material for determining the form of the orbit. The method of measurement is that described in the *Annals*, **26**, p. 236. The uncorrected positions of the four images referred to the first plate of August 16 as an origin are for x, $0.0''$, $+1.2''$, $+33.6''$, and $+71.8''$; for y, $0.0''$, $-1.7''$, $-19.8''$, and $-42.1''$; the corresponding Greenwich mean times are $12^h\ 16^m$, $14^h\ 18^m$, $12^h\ 56^m$, and $13^h\ 12^m$. Correcting for the motion of Saturn, the relative motion with references to that body is in x, $0.0''$, $-2.4''$, $-10.7''$, and $-22.0''$; in y, $0.0''$, $+0.1''$, $+2.4''$, and $+2.9''$. It appears from this that the apparent motion is about $10.4''$ a day, at a distance of $1480''$. A computation shows that if the orbit is circular, the period must be either 4200 or 490 days, according as the satellite is near conjunction or elongation. These values may be greatly altered if the orbit is elliptical. Since the interval of time between the first and last photographs on which the satellite appears is only two days,

it is impossible to predict its position with accuracy. It is probable that its position angle from Saturn now lies between 280° and 290°, and its distance between 20′ and 30′. These uncertainties will probably be greatly diminished from measures of plates of Saturn taken in Arequipa on September 15, 16, and 17, 1898, which for some unexplained reason have not yet been received in Cambridge.

The direction of the motion, which is nearly towards Saturn, shows that the apparent orbit is a very elongated ellipse, and that it lies nearly in the plane of the ecliptic. Professor Asaph Hall has pointed out that this is to be expected in a body so distant from Saturn. The attraction of the latter only slightly exceeds that of the Sun. Hyperion appears as a conspicuous object on all four of the plates, and the new satellite appears about a magnitude and a half fainter on each. The approximate magnitude is therefore about 15.5. As seen from Saturn, it would appear as a faint star of about the sixth magnitude. Assuming that its reflecting power is the same as that of Titan, its diameter would be about two hundred miles. It will, therefore, be noticed that while it is probably the faintest body yet found in the solar system, it is also the largest discovered since the inner satellites of Uranus in 1851. The last discovery of a satellite of Saturn was made in September 1848, by Professor William C. Bond, then director of this Observatory, and his son, Professor George P. Bond. The satellite Hyperion was seen by the son on September 16 and 18, but its true character was first recognized on September 19, when its position was measured by both father and son (see *Annals*, 2, p. 12). Soon after, it was discovered independently by Lassell at Liverpool.

Professor William H. Pickering, as the discoverer, suggests that the name Phoebe, a sister of Saturn, be given to the new satellite. Three of the satellites, Tethys, Dione, and Rhea have already been named for Saturn's sisters, and two, Hyperion and Iapetus, for his brothers.

EDWARD C. PICKERING.

April 10, 1899.

CHANGE OF ADDRESS.

I regret to state that Professor F. L. O. Wadsworth, who has been connected with the Yerkes Observatory from the beginning of its work, has resigned his position on the staff. His present address is 283 Lincoln Avenue, Bellevue, Pa., where all letters intended for him should be addressed.

G. E. H.

Reviews.

Harper's Scientific Memoirs, edited by Joseph S. Ames. No. 1. *The Free Expansion of Gases;* 8vo. pp. v + 106. No. 2. *Prismatic and Diffraction Spectra;* 8vo. pp. v + 68. Harper & Brothers, New York, 1898.

These two small volumes are the forerunners of a series of classic works in a variety of different fields of physical research, of which eight more volumes are already announced. A number of the papers to be thus gathered together, are translations from foreign languages, and the series may be regarded as a very modest beginning in English, of what has been more extensively done in German in the Ostwald reprints of classic scientific memoirs.

In addition to the service done by the editors in selecting and bringing together from different languages and times, in convenient form, the fundamental researches along a number of lines, they render available the original papers, which in some cases are now with difficulty accessible to those who would gain profit from reading them.

While it may be taken for granted that nearly all English-speaking physicists at the present time are able to read both French and German, yet the number who can read these languages as easily as their own, is relatively small; and most would prefer trustworthy translations, except perhaps in the narrow field where the student in question may himself be working.

The series cannot fail to accomplish a wider general knowledge, and a stimulated interest, among physical readers in the subjects treated, and it is a matter for congratulation that an American publisher has consented to do this service for English-speaking physicists; and that the selection of matter, and the editing of it, has fallen into such able hands. It is sincerely to be hoped that the narrow limits at present given to the series may be extended to include a greater number of special fields of research.

Of the two volumes already in hand, the "Memoirs on Prismatic and Diffraction Spectra" possesses the greater interest for spectroscopists. This volume contains the translation of three memoirs, and a short biography of Joseph von Fraunhofer, together with a fragment

of a memoir by William Hyde Wollaston and his biography, shortened down to a single six-line paragraph. The work ends with a bibliography of the more important contributions to spectroscopy, and an index.

The first of the Fraunhofer memoirs bears the translated title, "The Determination of the Refractive and the Dispersive Power of Different Kinds of Glass, with Reference to the Perfection of Achromatic Telescopes," and was originally published in the *Denkschriften der königlichen Akademie der Wissenschaften zu München*, V, pp. 193–226, 1817. The paper deals with the discovery of fixed bands in flame spectra, from which Fraunhofer passed directly to the use of sunlight, and the discovery of the lines in the solar spectrum which bear his name. These he found duplicated in the spectrum of Venus. He next turned his attention to Sirius, and in spite of the small amount of light which he was able to gather with an objective aperture of barely 3 cms, he says, "I have seen with certainty in the spectrum of Sirius three broad bands which appear to have no connection with those of sunlight; one of these bands is in the green, two are in the blue. In the spectra of other fixed stars of the first magnitude one can recognize bands; yet these stars, with respect to these bands, seem to differ among themselves." Thus was the science of stellar spectroscopy born into the world. Fraunhofer further observed that the refractive index of the bright band in the yellow of the lamp flame spectrum was exactly the same as that for the D line in the solar spectrum; and when seen with a very narrow slit, was a double line. He also describes the appearance of the electric spark spectrum.

The second memoir, on "A New Modification of Light by the Mutual Influence of the Diffraction of the Rays, and the Laws of this Modification," published likewise in the *Denkschriften*, VIII, pp. 1–76, deals with the laws of the diffraction of light through a single opening, and later, takes up the mutual action of a great number of diffracted beams. Here the description is given of the construction and study of the first wire diffraction gratings, and of the grating spectrum.

Following this, the third memoir gives a "Short Account of the Results of New Experiments on the Laws of Light, and Their Theory." The original paper was read before the Munich Academy in 1823, and subsequently published in Gilbert's *Annalen*. In this memoir a description is given of plane diffraction gratings ruled on glass, the exact law of the dispersion for normal and oblique incidence is enun-

ciated, and the effect of the form and width of the ruled lines upon the spectrum is discussed, together with the effect of unevennesses, and of periodic errors. Fraunhofer then proceeds directly from the laws he has developed from Young's theory of interference, to make the first measurements of the wave-lengths of the principal solar lines, in terms of the Paris inch.

The memoir closes with an addendum, describing the results of further observations, with an objective of four inches aperture, of lunar, planetary and stellar spectra. Fraunhofer finds that the Sun, Moon, Mars and Venus give similar spectra. He observes a resemblance between the spectrum of Sirius and that of Castor ; finds that Capella gives the same fixed lines as are found in sunlight ; and that the spectrum of Betelgeux contains more lines than the spectra of the planets, but some of them are in the same positions as prominent lines in the planetary spectra.

Many astronomers and astrophysicists will surely sympathize with this complaint which Fraunhofer makes in a footnote, "The further one goes with these experiments so much the wider becomes the field which offers itself for new investigations. It is greatly to be regretted that they can be repeated so seldom by anyone, owing to the fact that they demand very large, and, in part, expensive apparatus, and also a great deal of time. The fact that the sky must be most favorable makes one lose more time than would be believed, perhaps ; which I feel all the more because the demands of business leave me only a few definite days in the month which are free for these investigations."

The translation of the memoirs, throughout, has been faithfully and accurately done, and the relative simplicity of Fraunhofer's German, the clearness of his thought and expression, have been most admirably reproduced in English. The editor makes frequent omissions from unimportant parts of the memoirs, presumably to avoid exceeding a certain number of pages, and also to save his readers' patience ; for it must be remembered that Fraunhofer first undertook the studies here described purely for the purpose of aiding himself in the manufacture of better achromatic lenses, and consistent with this purpose, made many laborious studies of different kinds and qualities of glass, which are not of the same scientific importance as much of his other work. This editor's privilege has, however, been exercised to the extent of omitting several diagrams and explanations of apparatus. From the reader's point of view less can be said in favor of omissions of the latter sort. E. F. N.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

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Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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